



AN APPROXIMATION TO LATERAL EARTH PRESSURES FOR K_0 CONDITION

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ABSTRACT

In this study, the determination of lateral earth pressures of soils or K_0 parameter is considered. For this effect, the deformation and the variations in the shear stresses of the soils placed in an oedometer set up were investigated. Based on this data, a general method which can be used in the calculation of lateral pressures of soils has been proposed. The study was carried out on a cohesive soil having two different group symbol and sandy soils with different relative densities. The lateral pressure values measured by thin wall oedometer technique are in very good agreement with those obtained by calculation. In conclusion, lateral earth pressures or the K_0 values are depend upon the distribution of the samples, their relative density and consistancy, the magnitude of the pre-consolidation pressure. The proposed method is a simple and economic technique as regards to the approximation and experimentation.

Key Words : Lateral earth pressure at rest, Oedometer test, Consolidation, Shear stresses of soils in oedometer

K_0 KOŞULLARINDA YANAL TOPRAK BASINÇLARI İÇİN YAKLAŞIM

ÖZET

Bu çalışmada zeminlerin yanıl toprak basınçları veya K_0 parametresinin tayini ele alınmıştır. Bu etki için bir odömetre aletine konulan zemin örneğine ait kayma gerilemelerinin değışimi ve deformasyonları incelenmiştir. Bu verilere dayanarak yanıl toprak basınçlarının hesaplanabildiđi genel bir yöntem ileri sürölmüştür. Araştırmalar, farklı iki grup sembolüne sahip kohezif zemin ile farklı rölatif sıklıktaki kumlu zeminler üzerinde sürdürölmüştür. İnce cidarlı odömetre tekniđi ile ölçölen ve hesaplanan yanıl basınç değeri; birbirlerine oldukça yakındır. Sonuç olarak; yanıl toprak basınçları veya K_0 değeri, zemin örneklerinin bozulup bozulmadığına; örneklerin rölatif sıklık ve konsistansına, ön konsolidasyon basıncının büyüklüğüne bađlıdır. İleri sürölen yöntem, hem ekonomik hem de basit bir yöntemdir.

Anahtar Kelimeler : Sükunetteki toprak basıncı, Odömetre deneyi, Konsolidasyon, Odömetredeki zeminlerin kayma gerilmesi

1. INTRODUCTION

At most of the geotechnical problems the determination of lateral earth pressure is important for the determination of design parameter K_0 . The determination of this parameter by a consolidation test has a very big importance to find its changes with depth in boring and in making a more realistic design.

The lateral earth pressures caused by a particular soil depends on so many factors such as its history, consolidation condition, void ratio, porosity and structure (Bishop and Henkel, 1962). K_0 were experimentally found to have a value around 0.40 for sandy soils and 0.70 for cohesive ones. Also, there are many studies proving that initial void ratio and the plasticity of the soil have an important effect on K_0 (Kumbasar, 1956).

In practice, the lateral earth pressure is determined either directly by in-situ methods or indirectly by laboratory techniques. In laboratory techniques, sophisticated equipment such as Tri-axial test or specially equipped oedometers are used for the determination of K_0 (Abdulhamid and Krizek, 1976; Menzies et al., 1977; Edil and Dhowian, 1981).

Another alternative to determine the K_0 values is the use of theoretical and empirical relationships. Since the problem is an hyperstatical one, these methods require the knowledge of additional soil parameters such as internal friction angle (ϕ), over consolidation ratio and plasticity index (Krizek and Abdulhamid, 1967; Abdulhamid and Krizek, 1976).

Furthermore, in many of the laboratory techniques, they are trying to determine the fictitious parameters such as Poisson's ratio and modulus of elasticity of soils inspite of the fact that soils are not elastic materials. K_0 is defined as follows using the Hook laws by assuming that the lateral deformation of an elastic body can be neglected ($\epsilon_x = \epsilon_y = 0$) and the body is consolidated under the principal vertical stress:

$$K_0 = \frac{\mu}{\mu - 1} \quad (1)$$

Here, μ is the Poisson's ratio and K_0 is the coefficient of lateral earth pressure at rest (Lambe and Whitman, 1979).

In addition to those, the frictional forces in the laboratory techniques used to determine the K_0 value, were found to cause 12-22 % changes in the vertical stresses applied on the surface of the samples in the case of distributed clays and 15 % in the case of the undistributed samples. These variations can sometimes go up to the level of 40 % in the stresses transferred to the lower surface of the samples. (Manden, 1969; Taylor, 1942). Furthermore, Terzaghi demonstrated that in the case where lateral deformation was larger than the level of 10^{-3} caused the appearance of passive or active conditions and the presence of equilibrium in a very limited region of deformation (Sağlamer, 1972; Bedişkan, 1993).

The following definition given in Eq. (2) becomes much more meaningful than the previous expression (1) as regards to minimize the disadvantages in the transfer of stresses and include the equilibrium in the elastic and plastic region of the deformation, of any body (Andrews and El-Shoby, 1973). Also the presence of two dimensional stresses in oedometer

tests and the use of thin soil samples support this thesis (Sağlamer, 1972).

$$K_0 = \frac{\sigma_h}{\sigma_v} \quad (2)$$

Where σ_h and σ_v represent the lateral earth and vertical pressures, respectively.

The reasons have been mentioned up to now bring the use of classical consolidation tests, where thin soil samples are used, in the determination of lateral earth pressure and K_0 values. However, a method of calculation of shear stress variations and lateral pressures in a soil sample placed into a conventional oedometer has not been known yet. Therefore, the main goals of our study can briefly be outlined as follows:

- i) To determine the variations of stresses in a sample placed in an conventional oedometer,
- ii) To calculate lateral earth pressures related to vertical stress variations and compare these calculated values with those determined with the conventional test methods,
- iii) To develop a simple and a cost effective method for K_0 determination and the other soil parameters.

2. MATERIALS AND METHODS

In this study, the lateral pressures of clayey soils were determined by thin wall oedometer test technique (Ertekin, 1991; Bedişkan, 1993) and the lateral pressures of sandy soils were found by the use of oedometer developed by Sağlamer in 1972. The measurements were carried out directly by electrical strain gauges mounted on the wall of the oedometer.

A normally consolidated condition was created initially for the cohesive samples used in the experiments (specimen 1 and 2) by "Sullary Consolidation method" and the load was discharged by consolidating the soil under 1600 kpa (see Table 1). The discharged soil was regarded as over consolidated soil. The undistributed samples having two different group symbols were also subjected to the consolidation tests and its lateral pressure variation is measured in order to determine the situation in undisturbed soils (Chikhouni, 1991; Bedişkan, 1993). The undisturbed cohesive soils used in the experiments were over consolidated soils which index properties are tabulated in Table 1. The values related to measured and calculated lateral pressures are given

in Table 3 and Table 4 in order to set up an example.

Table 1. Index Properties of Cohesive Soils (Bedişkan, 1993)

Disturbed and Remoulded Clay Samples (Specimen 1 and 2)			Undisturbed Soil Samples	Specimen 3	Specimen 4
Liquid limit (L.L)	63 %		Boring depth	7.5	14.0 m
Plastic limit (PL)	27 %		Liquid limit (L.L)	65 %	39 %
Plasticity index (IP)	36 %		Plastic limit (PL)	23 %	22 %
Specific gravity (GS)	2.715		Plasticity index (IP)	42 %	17 %
Finer than No. 200	53 %		Natural water content	29.9 %	14.4 %
Retain on No. 40	22 %		SPT values	32	>50
Group symbol	CH		Amount passed thr. No. 200	85 %	61 %
			Group symbol	CH	CL

There were fine, medium and coarse sands taken from different sites for the determination of the lateral stresses related to sandy soils. Their physical properties are given in Table 2. The data belonging

to dense Kilyos sand with a relative density of $D_r = 0.89$ is listed in Table 5 in order to give an example of calculations and measurements.

Table 2. Physical Properties of Sandy Soils (Sağlam, 1972)

The Name of the Sand	The Type of the Sand	Density (g/cm ³)	Uniformity, $U D_{60}/D_{10}$	Max. Void Ratio, e_{max}	Min. Void Ratio, e_{min}	Spherity	Mineralogy
Kilyos	Fine	2.72	1.25	0.81	0.45	0.75	45 % Quartz 50 % Calcite + Aragonite 5 % Magnetite
Ayvalık	Medium	2.64	1.30	0.91	0.59	0.60	80 % Quartz 19 % Calcite + Aragonite 1 % Magnetite
Yahkoy	Coarse	2.66	1.00	0.67	0.44	0.70	99.9 % Quartz 0.1 % Magnetite
Synthetic	Coarse	1.18	1.00	0.69	0.59	1.00	100 % Plexyglass

3. RESULTS AND DISCUSSION

One can easily see from Table 3, Table 4 and Table 5 given for comparative purposes that the calculated values are in good agreement with the measured counterparts. In addition, since the standard deviation of the calculated values, at a probability level of 95 %, $t_{0.95}$, is less than 2.02, they can be accepted as the elements of the same population. In the experiments, the lateral stresses of the soil and

its vertical displacements under a certain vertical pressure were measured. The lateral stresses measured under a constant vertical pressure after dissipation of pore water pressure were accepted as the effective stresses (see Table 3 and 4).

The second specimen having the same group symbol with the first one was subjected to preloading under 1600 kpa and an over consolidated soil sample was obtained after the discharge of the load (see table 4).

Table 3 The comparison of the Thin Wall Oedometer Test Results and the Computed Values of Lateral Earth Pressures (Specimen 1 or Remoulded Sample - Bedişkan, 1993)

Sample Height Initial : 6.00 cm Final : 3.98 cm		Group Symbol : CH			Water Content Initial : 44 % Final : 16.9 %		Mean Sq. Error $M = \pm 1.011$ kpa T Value (95 % Prob) = 2.02 (The σ_H Values Are in the Same Population.)		
Type of Loading	Over Cons. Ratio (O.C.R) (Kpa/Kpa)	App. Vertst Ress, σ_v (Kpa)	Measu. Hor. Stress, σ_H (Kpa)	Void Ratio, E (%)	Strain $\epsilon_\tau = \frac{\Delta e}{1+e_0}$ Mm/Mm	Tang. of Def Ang. $\tan 2\phi = \frac{2-e_\tau}{2e_\tau}$	Sin. of Def Ang. $\sin 2\phi$	α Par.Of Shear Stress $\alpha = \left(\frac{\sigma - \sigma'}{2\tau'} \right)$	Computed Hor.Stress σ_H (Kpa)
Virgine Loading	Norm Cons	50.0	36.60	1.20 1.00	0.09091	10.49989	0.995495	1.0306	36.61
"	"	100.0	74.50	0.85	0.15909	5.78575	0.985390	0.9446	74.53
"	"	200.0	144.60	0.70	0.22727	3.90005	0.968665	0.9806	145.76
"	"	400.0	294.60	0.55	0.30000	2.83333	0.942990	0.8801	293.67
"	"	600.0	444.70	0.46	0.34091	2.43333	0.924940	0.8327	443.33

Table 4. The comparison of the Thin Wall Oedometer Test Results With the Computed Values of Lateral Earth Pressures (Specimen 2 or Remoulded Sample - Bedişkan, 1993)

Sample height Initial : 6.00cm Final : 3.98cm			Group Symbol : CH m = ± 1.33 kpa P _c = 1600 kpa t _{0.95} = 2.02 (The σ_h are in the Same Population.)					Water Content Initial : 11.4 % Final : 9.1 %	
Type of Loading	Overcons. Ratio (O.C.R) (kpa/kpa)	App. Vertical Stress, σ_v (kpa)	Meas. Horiz. Stress, σ_h (kpa)	Void Ratio, e (%)	Strain $\epsilon_r = \frac{\Delta e}{1+e_0}$ mm/mm	Tang. of Def Ang. $\tan 2\phi = \frac{2-\epsilon_r}{2\epsilon_r}$	Sin. of Def Ang. $\sin 2\phi$	α Par. of Shear Stress $\alpha = \left(\frac{\sigma_v - \sigma_h}{2\tau_\eta} \right) \times \sin 2\phi$	Computed Hor. Stress σ_h (kpa)
Over Consol.	32	50	203	38.0	0.03139	31.35728	0.999492	-12.0895	201.87
"	16	100	241	33.2	0.03600	27.27778	0.999329	-5.5599	242.01
"	8	200	283	31.5	0.04943	19.73063	0.998718	-1.6271	284.11
"	4	400	371	29.1	0.06894	14.00537	0.997461	0.2818	369.12
"	2	800	633	25.5	0.09960	9.54016	0.994551	0.7993	633.09
Virgine Loading	1	1600	1167	18.5	0.16456	5.57681	0.984301	0.9991	-----

The porous stones of an oedometer test equipment can be accepted as a rigid body. Therefore, the pressure on the soil specimen is higher at the edges and lower in the middle in a clayey soil. The situation in sandy soils, on the other hand, is just the opposite. There is also in question of arching effect

at the walls of oedometer. When friction factor is taken into account, in addition to all these, lateral σ_h and vertical σ_v stresses are seen to be very rough values, but they give satisfactory results for the engineering applications (Hardy, 1983).

Table 5. Oedometer Test Results of Sandy Soil Sample of Kilyos (Densed Soil Sample -Sağlam, 1972)

Initial Volume: 158.94 cm ³ ; $\gamma_d = 1.82$ gr/cm ³ ; $e_0 = 0.49$; $D_r = 0.89$; Bore No: Kilyos 5-5							
Type of Loading	App. Ver Stress, σ_v (kgf/cm ²)	Meas. Hor. Stress, σ_h (kgf/cm ²)	Volumetric Strain $\epsilon_r = \frac{\Delta e}{1+e_0}$ (mm/mm)	Tang. of Def Ang. $\tan 2\phi = \frac{2-\epsilon_r}{2\epsilon_r}$	Sin. of Def Ang. $\sin 2\phi$	α Par. of Shear Stress $\alpha = \left(\frac{\sigma_v - \sigma_h}{2\tau_\eta} \right) \sin 2\phi$	Computed Hor. Stress σ_h (kgf/cm ²)
Virgine loading	1.00	0.35	0.0034	293.61765	0.999994	2.5958	0.3616
"	1.96	0.70	0.0042	237.59524	0.999991	2.5672	0.7162
"	2.90	1.05	0.0049	203.58163	0.999988	2.5468	1.0552
"	3.81	1.40	0.0055	181.31818	0.999985	2.5251	1.4094
"	4.71	1.75	0.0059	168.99152	0.999982	2.5080	1.7558
"	5.60	2.11	0.0063	158.23016	0.999980	2.4868	2.1042
"	6.49	2.46	0.0069	144.42754	0.999976	2.4772	2.4720
"	7.37	2.84	0.0074	134.63514	0.999972	2.4518	2.8354

In addition to all these, swelling and collapsible properties of soils have also important effects on their stress-strain relationship, as well as consolidation situation. Because, when the specimen is subjected to water after a certain pressure, particularly the clayey soils have higher plasticity and having a group symbol of CH, show swelling and this in turn effects the stress-strain relationship. Also the history of the soil and the mineral contains give its collapsible property when subjected to water under stress.

Another factor which effects the stress-strain relationship of the soil in the oedometer is that the prevention of lateral displacement causes the

deformations taking place to be in the volumetric strain. Furthermore, the soil is a non-elastic material and the shear stresses have a marked influence on the stress condition in the soil. That is why, the shear stresses place in the soil in an oedometer should carefully be taken into account. In conclusion, the stress-strain relationships in the oedometer should be reevaluated.

In order to define the shear strain of the soil sample in an oedometer, the model proposed in Figure 1.a. can be used by considering shear strains of the elementary cylinders. According to this figure, $\Delta z/1 = \epsilon_1$ represents the volumetric strain since the lateral displacements are prevented and therefore the

lateral deformations are not shown in the figure. When the body in Figure 1.a is deformed, point B comes to point C and the diagonal AB takes the shape of AC. Therefore an arbitrary axis of ξ passing through AC and η is vertical to this axis can be taken into account and the corresponding shear strain is represented as $\gamma_{\xi\eta}$. When the tangent of angle \widehat{OAC} is considered and $\gamma_{\xi\eta}$ is assumed to be a very small angle then the following relationships can be written:

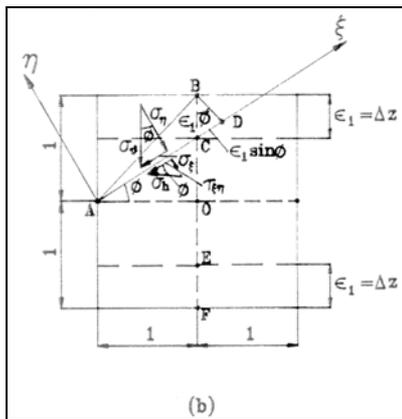
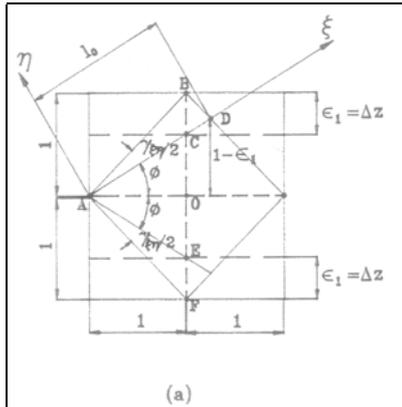


Figure 1. Determination of shear angle and strain

$$\tan \widehat{OAC} = \tan \left(\frac{\pi}{4} - \frac{\gamma_{\xi\eta}}{2} \right) = \frac{OC}{OA} = \frac{1 - \epsilon_1}{1}$$

$$\tan \left(\frac{\pi}{4} - \frac{\gamma_{\xi\eta}}{2} \right) = \frac{\tan \frac{\pi}{4} - \tan \frac{\gamma_{\xi\eta}}{2}}{1 + \tan \frac{\pi}{4} \times \tan \frac{\gamma_{\xi\eta}}{2}} \quad \text{and} \quad \tan \frac{\gamma_{\xi\eta}}{2} \cong \frac{\gamma_{\xi\eta}}{2} \quad (3)$$

$$\gamma_{\xi\eta} = \frac{2\epsilon_1}{2 - \epsilon_1}$$

On the other hand, when the projection of deformation ϵ_1 on the axis ξ is taken and called as ϵ_ξ , then the following equation is written (Figure 1.b)

$$\epsilon_\xi = \epsilon_1 \sin \phi \quad (4)$$

Where ϕ shows the angle which the cross section makes with the lateral axis after the settlement.

Let us investigate the value of ϕ in terms of volumetric strain ϵ_1 . When the tangent of angle \widehat{BAF} is written according to Figure 1.a then the followings can be written.

$$\tan(2\phi + \tan \gamma_{\xi\eta}) = \frac{\tan 2\phi + \tan \gamma_{\xi\eta}}{1 - \tan 2\phi \times \tan \gamma_{\xi\eta}} = \tan \left(\frac{\pi}{2} \right) = \infty \quad (5)$$

Since $\tan 2\phi$ and $\tan \gamma_{\xi\eta}$ have finite values, the equation (5) can only be infinite when $\tan 2\phi \times \tan \gamma_{\xi\eta} = 1$. Therefore, taking $\tan \gamma_{\xi\eta} \cong \gamma_{\xi\eta}$ the followings can be given.

$$\tan 2\phi = \frac{1}{\gamma_{\xi\eta}} \quad (6.a)$$

If the value of $\gamma_{\xi\eta}$ is substituted into this equality from equation (3), then

$$\tan 2\phi = \frac{2 - \epsilon_1}{2\epsilon_1} \quad (6.b)$$

The stress-strain relationship of a soil in an oedometer is a concave curve. Under these circumstances there is no linearity. The shear modulus of the soil is both depend on the stresses and the deformations in the soil ($G = G(\sigma_v, \sigma_r)$). Since the increase of the vertical stresses in the soil in an oedometer cause the strain and the shear stresses have different values, therefore the following relations can be given;

$$\tau_{\xi\eta} = G(\sigma_v, \epsilon_\xi) \gamma_{\xi\eta} \quad \text{and} \quad \tau_{\xi\eta} = G(\sigma_v, \epsilon_\xi) \left(\frac{2\epsilon_1}{2 - \epsilon_1} \right) \quad (7)$$

Where $\tau_{\xi\eta}$ is the shear stress determined according to ξ and η plane (F/L^2) and $\gamma_{\xi\eta}$ is the shear strain at the same plane and G is the shear modulus (F/L^2).

In an oedometer test, the soil specimen under a constant pressure conserve its equilibrium after the settlement process is completed. That is why, the equilibrium of the elementary cylinders given in Figures 1.a and 1.b should be examined. Since the equilibrium are established at every point of the soil, it will be much more logical to examine the equilibrium of the elementary cylinder taken along the ξ axis. Let us take cylinder having a very small

radius which axis coincides with the ξ axis. The part of this cylinder adjacent to the oedometer wall is inclined and formed an ellipse.

The axis of this ellipse are r and a as indicated in Figure 2. If the equilibrium of this elementary cylinder is examined, then the equation of equilibrium is written along the cylinder axis, the followings may be written.

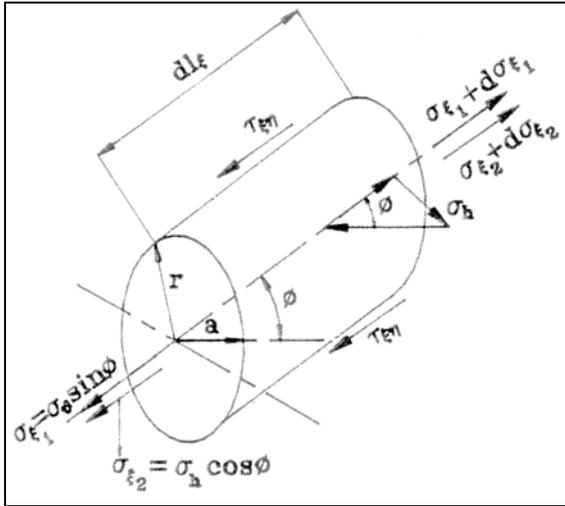


Figure 2. Equilibrium of elementary soil sample along ξ axes

$$\pi a r (\sigma_{\xi_1} + \sigma_{\xi_2}) - \pi a r (\sigma_{\xi_1} + d\sigma_{\xi_1} + \sigma_{\xi_2} + d\sigma_{\xi_2}) + 2\pi r \tau_{\xi\eta} dl_{\xi} = 0$$

$$a(d\sigma_{\xi_1} + d\sigma_{\xi_2}) = 2\tau_{\xi\eta} dl_{\xi} \quad (8a)$$

Here, the stresses are assumed to change from point to point, when angle ϕ and the cross section are kept constant.

One can see that $\varepsilon_{\xi} = l_{\xi} - l_0 / l_0 = 1 - l_{\xi} / l_0$ and $dl_{\xi} = -l_0 d\varepsilon_{\xi}$ from Figure 1.a and $\varepsilon_{\xi} = \varepsilon_1 \sin \phi$ from equation 4. Substituting these into 8.a it turns into

$$\frac{a}{l_0} (d\sigma_{\xi_1} + d\sigma_{\xi_2}) = -2\tau_{\xi\eta} d\varepsilon_{\xi} \quad (8.b)$$

In which $d\sigma_{\xi_1} = d\sigma_v \sin \phi$ and $d\sigma_{\xi_2} = d\sigma_h \cos \phi$ are componets. When they are substituted into (Figure 1.b and Figure 2) Eq (8.b), then

$$\frac{a}{l_0} (d\sigma_v \sin \phi + d\sigma_h \cos \phi) = -2\tau_{\xi\eta} d\varepsilon_1 \sin \phi$$

Is obtained. When dividing both side with $\sin \phi$ and substituting the value of $\tau_{\xi\eta}$ from (7) and integrating, then the equation gives;

$$\left(\frac{a}{l_0}\right) \left(\int_0^{\sigma_v} d\sigma_v + \cot \phi \int_0^{\sigma_h} d\sigma_h \right) = -4G \int_0^{\varepsilon_1} \frac{\varepsilon_1 d\varepsilon_1}{2 - \varepsilon_1} \quad (9.a)$$

Considering the right hand side of 9.a, the following can be written by making the conversions of $-\varepsilon_1 = \varepsilon_1$ and $d\varepsilon_1 = -d\varepsilon_1$ in order to carry out the calculation using positive values.

$$-\int_0^{\varepsilon_1} \frac{\varepsilon_1 d\varepsilon_1}{2 - \varepsilon_1} = -\int_0^{\varepsilon_1} \frac{\varepsilon_1 d\varepsilon_1}{2 + \varepsilon_1} = \int_0^{\varepsilon_1} \left(1 - \frac{2}{2 + \varepsilon_1}\right) d\varepsilon_1 = \varepsilon_1 - 2 \ln(2 + \varepsilon_1) + 2 \ln 2$$

$$\int_0^{\varepsilon_1} \frac{\varepsilon_1 d\varepsilon_1}{2 + \varepsilon_1} = \varepsilon_1 - \ln\left(\frac{2 + \varepsilon_1}{2}\right)^2$$

When the second term of right hand side in this equation is taken into account, it is seen that the deformations are equal to their absolute values. If $\varepsilon_1 = -\varepsilon_1$, then one can see from the experimental values that

$$\varepsilon_1 = -\ln\left(\frac{2 - \varepsilon_1}{2}\right)^2$$

Therefore the integral 9.a becomes

$$\left(\frac{a}{l_0}\right) \sigma_v \left(1 + \frac{\sigma_h}{\sigma_v} \cot \phi\right) = 4Gx2\varepsilon_1 \quad (9.b)$$

If we recall the definition of K_0 in equation (2) then one obtains

$$G = \frac{1}{8} \left[\frac{\sigma_v \left(\frac{a}{l_0}\right) (1 + K_0 \cot \phi)}{\varepsilon_1} \right] \quad (9.c)$$

The term $\alpha = (a/l_0) (1 + K_0 \cot \phi)$ is depend upon the type of the soil and therefore, the angle between the cross section of the soil and lateral line, and (a/l_0) is a parameter depend upon small axis of the ellipse formed at the oedometer wall, the diagonal l_0 consequently to the type of the soil. When the value of shear modulus given by (9.c) is substituted in (7) together with the value of shear strain given by (3) then the value of $\tau_{\xi\eta}$ can be given by

$$\tau_{\xi\eta} = \frac{\alpha}{4} \left(\frac{\sigma_v}{2 - \varepsilon_1}\right) = \alpha \tau \quad (10)$$

Where ε_1 represents the absolute value of deformations.

The loading of the soil sample in the oedometer causes the angle $\gamma_{\xi\eta}$ to change depending upon ε_1 and the rotation of the axis in the Mohr plane. The change of $\gamma_{\xi\eta}$ results the change in $\tau_{\xi\eta}$ according to Eq.(7) and in shear modulus according to (9.c). In conclusion, the shear modulus of the soils are not constant according to (9.c) and depend upon the lateral stress or σ_v and the values of volumetric strain ε_1 due to its dependence upon the type of soil (in relation to α parameter)

On the other hand, if Figure 1.b is reconsulted, there are shear stresses $\tau_{\xi\eta}$ on the plane ξ . There are normal stresses σ_ξ in the vertical direction to axis ξ . Under these circumstances, the maximum stress effecting on the soil element will be the principal stress as σ_v . The stress condition given in Figure 1 can be represented by the Mohr circle given in Figure 3. When one take the projection according to Figure 2 and use the Mohr circle, then one can write the following equations for an element having a unit width (Figure 3).

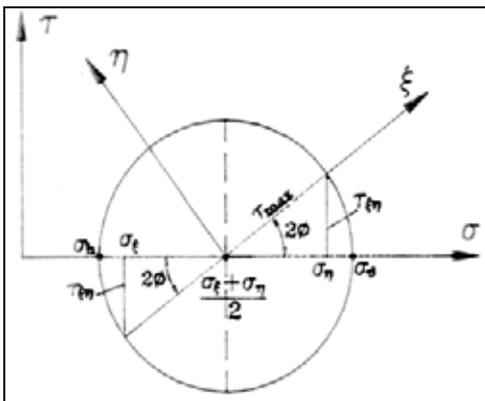


Figure 3. The rotation of axes in the Mohr plane

$$\tan 2\phi = \frac{2\tau_{\xi\eta}}{\sigma_\eta - \sigma_\xi} \quad (11.a)$$

$$\sigma_\xi = \sigma_v \sin \phi + \sigma_h \cos \phi - \tau_{\xi\eta} = \sigma_v \sin \phi \left(1 + K_0 \cot \phi - \frac{\tau_{\xi\eta}}{\sigma_v} \csc \phi \right)$$

$$\sigma_\xi \cong \sigma_v \sin \phi \quad (11.b)$$

$$\sigma_\eta \cong \sigma_\xi + \frac{2\tau_{\xi\eta}}{\tan 2\phi} \quad (11.c)$$

In normal consolidated soils $K_0 < 1$. Also angle ϕ is around 45° , then $\cot \phi \cong 1$. The value of $\tau_{\xi\eta}$ is much smaller than σ_v , the ratio of $\tau_{\xi\eta} / \sigma_v$ is smaller than unity. Also $\csc \phi$ has a negative value close to $\sqrt{2}$ since angle ϕ is about 45° . Therefore, the value of $K_0 \cot \phi - (\tau_{\xi\eta} / \sigma_v) \csc \phi$ is very close to zero. As a

result, taking the value of σ_ξ as in (11.b) before the iteration and carrying out the iteration of over consolidated soils as in normally consolidated soils using (11.b), do not cause a significant error since the problem is a hyperstatic and definitely necessitates an iteration. Again one can write the following equations for the principal stresses σ_v and σ_h using Figure 3 and considering Eq.10.

$$\sin 2\phi = \frac{2\tau_{\xi\eta}}{\sigma_v - \sigma_h} = \frac{2\alpha\tau}{\sigma_v - \sigma_h} \quad (12.a)$$

in which $\tau = \frac{1}{4} \left(\frac{\sigma_v}{2 - \varepsilon_1} \right)$ (from Eq.10)

$$\alpha = \left(\frac{\sigma_v - \sigma_h}{2\tau} \right) \sin 2\phi \quad (12.b)$$

The Equation 12.b can be used to determine the values α , using the experimentally determined σ_v and σ_h values (see the given tables) It is also possible to investigate the variation of α with the vertical pressures σ_v . Such a variation is given in Figure 4 .

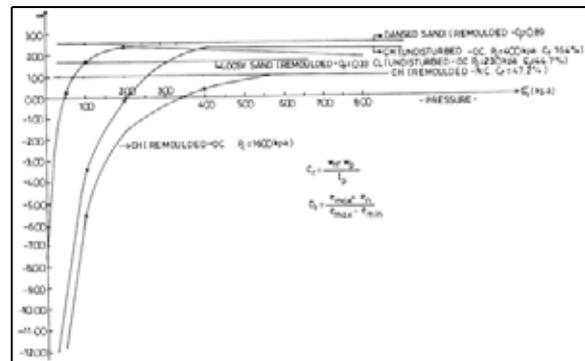


Figure 4. The relationship between α and vertical stress

When this figure is examined, one can see that there is a very good correlation between vertical pressures σ_v and α . The figure also shows that $\sigma_v - \alpha$ curves have linearity for remoulded soils. This linearity shifts upwards for sandy soils parallel to the increase in relative density. The same behavior is also observed for cohesive soils and the linearity shifts upward in relation to relative consistency and preconsolidation pressure P_c .

For over consolidated and undisturbed soils, the curve becomes linear after the pre- consolidation pressure P_c and becomes parallel to the curve belonging to remoulded soil. This enables one to determine $\tau_{\xi\eta}$ from Eq. (10). After defining α from

Figure 4 and carry out interpolation for the values of the relative density and for relative consistence which remain between these limits.

The maximum shear stresses have special importance for the soils to gain shear strength. However the direction of maximum shear stresses is related to the principle axes of stress and strain. Therefore, the following equalities can be written using the Mohr Circle given in Figure 3 (See appendix III).

$$\tau_{\max} = \left[\left(\frac{\sigma_{\eta} - \sigma_{\xi}}{2} \right)^2 + \tau_{\xi\eta}^2 \right]^{1/2} = \frac{\sigma_v - \sigma_h}{2} \quad (13.a)$$

$$\sigma_h = \sigma_v - 2\tau_{\max} \quad (13.b)$$

$$\sigma_h = \sigma_v - \frac{\tau_{\xi\eta}}{\sin 2\phi} \quad (13.c)$$

Now, since σ_v and σ_h values are known, we can investigate the normal stresses of σ_{n1} and σ_{n2} shown in Figure 5.a acting on the AD section in Figure 1b and $\tau_{\xi\eta}$ stress can be examined. If one writes the equilibrium of the element given in Figure 5a in the direction of σ_{n1} vertical to AD and in the direction parallel to AD one ends up with the followings. (İnan, 1984) (see appendix III).

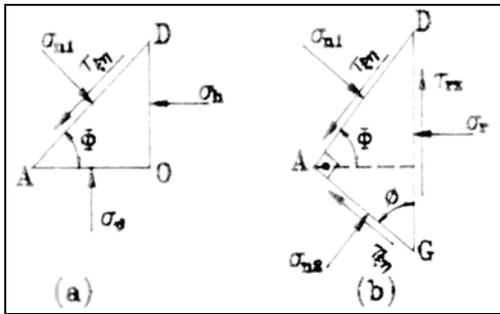


Figure 5. Two dimensional stress condition

$$\sigma_{n1} = \frac{\sigma_v + \sigma_h}{2} + \left(\frac{\sigma_v - \sigma_h}{2} \right) \cos 2\phi \quad (14.a)$$

$$\sigma_{n2} = \frac{\sigma_v + \sigma_h}{2} - \left(\frac{\sigma_v - \sigma_h}{2} \right) \cos 2\phi \quad (14.b)$$

$$\tau_{\xi\eta} = \left(\frac{\sigma_v - \sigma_h}{2} \right) \sin 2\phi \quad (14.c)$$

Here, the normal stress of σ_{n2} is the stress on the section rotated by $\pi/2$ as regard to AD. Under these condition, one can write

$$\tan 2\theta = \frac{2\tau_{\xi\eta}}{\sigma_{n1} - \sigma_{n2}} \quad (15)$$

By the help of Mohr Circle, in order to define the direction of the principle axes of stress and strain (see appendix III). This is supposed to give the same value with equation (6.b). In other words; $\tan 2\theta = \tan 2\phi$. Now the vertical and lateral stresses can be expressed as follows by using Figure 5.b. (İnan, 1984). (appendix III).

$$\sigma_z = \frac{\sigma_{n1} + \sigma_{n2}}{2} + \left(\frac{\sigma_{n1} - \sigma_{n2}}{2} \right) \cos 2\theta + \tau_{\xi\eta} \sin 2\theta \quad (16.a)$$

$$\sigma_r = \frac{\sigma_{n1} + \sigma_{n2}}{2} - \left(\frac{\sigma_{n1} - \sigma_{n2}}{2} \right) \cos 2\theta - \tau_{\xi\eta} \sin 2\theta \quad (16.b)$$

$$\tau_{rz} = \left(\frac{\sigma_{n1} - \sigma_{n2}}{2} \right) \sin 2\theta - \tau_{\xi\eta} \cos 2\theta \quad (16.c)$$

Since the vertical and lateral stresses are the principal stresses, then $\sigma_z = \sigma_v$,

$\sigma_r = \sigma_h$ and $\tau_{rz} = 0$. However, as the value of σ_v increases the σ_h values found from (13.b) and (13.c) deviates from experimental values. Here, $\tan 2\theta$ values are different from $\tan 2\phi$ values and they are to be corrected. The equation (15) can be written as (by the assumption that $\tan 2\theta = \tan 2\phi$) (see appendix III).

$$\left(\frac{\sigma_{n1} - \sigma_{n2}}{2} \right) \tan 2\phi - \tau_{\xi\eta} = 0 \quad (17)$$

Since the value to be obtained from this equation is different from zero, the initial value of $\tau_{\xi\eta}$ should be taken different from the first value. Since $\tau_{rz} = 0$ according to equation (16), $\sin 2\theta$ and $\cos 2\theta$ have great importance and they should not be changed. Therefore, the value of $\tau_{\xi\eta}$ is corrected by trial and error until $\sin 2\theta = \sin 2\phi$; $\cos 2\theta = \cos 2\phi$ and $\tan 2\theta \cong \tan 2\phi$. If the value found from equation (17) comes out very small then the calculation, then the procedure should be continued by small increments until $\sin 2\theta = \sin 2\phi$; $\cos 2\theta = \cos 2\phi$ (see appendix IV). Although the shear stresses in normally consolidated soils are positive, these values are negative until pre-consolidation pressure for over consolidated soils. The fact that the shear stresses are negative in sign indicates that the soil gains shear strength by pre loading. These gains in strength, the energy dissipation during discharging

and the inelastic behavior of soil cause the hysteresis loop. This results in the rotation of origin of plane and the principal axes of stress. The fact that the shear stresses are negative in sign do not contradict to the thesis of Skempton (1961) and Toğrol (1967) stating that in the case of a soil which becomes consolidated under the increasing loads, carries a lower vertical load as a result of the erosion of the upper layers there will be no changes in the lateral stresses. The lateral stresses are found to be larger than the vertical stresses especially in the over consolidated soil having the group symbol of CH. Figure 4 shows that the parallel shifts of the curves are related to the relative consistency and relative density. In conclusion, initial void ratios, consistency and the relative density of the soils and pre consolidation pressures have a marked effect on the shear stresses. The disturbance of the soil is another factor which effect the shear stresses and consequently the shear strength. It can also be said that the resulting shear stresses in the soil is related to the over consolidation ratio (O. C. R).

The volumetric strain values ε_v in over consolidated soils are found to be smaller than normally consolidated soils. This is related to the consistency and the relative density of the soil. For instance, while smaller strains are being observed in an unsaturated cohesive soils it is obtained that the strains are increasing relatively as the soil is given water. In this situation, The strains can be kept only possible when the vertical pressure of the soil is larger than the swelling pressure. In the case of the soil gained a strength due to the preconsolidation, the axes of the normal stresses σ and shear stresses τ in the Mohr Plane rotates. In addition, the fact that the lateral stresses are larger than vertical stresses indicates that the plasticity index of clayey soils have an important role. For example, in clayey soils having the group symbol of CH, the lateral stresses are larger than the vertical stresses for the small values of the preconsolidation pressure. In conclusion, it is obvious that the sensitivity, and the structure have a marked effect on the lateral stresses as much as the magnitude of preconsolidation pressure p_c .

4. CONCLUSION

The lateral stresses in a soil placed in an oedometer depend on previously acquired shear stresses and the strength as well as the consolidation situation, consistency or the relative density of the soil. In the case of cohesive soils; the plasticity of the soil play important role on the lateral and the shear stresses.

The shear strains in soils are not linear, $\gamma = 2\varepsilon_1$ as in an elastic body and they are the non-linear function of ε_1 as given in equation (3). The shear modulus, on the other hand, changes according to the effective stresses and strains. Therefore, the parameters such as Poisson's ratio and modulus of elasticity are of the fictitious in nature. In the final analysis of the definition of $K_0 = \sigma_h / \sigma_v$ much more meaningful than the description given by equation (1).

As seen from the tables, the calculated stresses are in the same population with the measured values. Therefore the proposed model provides economy as regards to experimental procedures.

The K_0 value of the soils can be found by the proposed method. The fact that linearity occurs after the preconsolidation pressure p_c in Figure 4 shows that the system is being normally loaded. Under these circumstances, the expression of $K_0 = 0.95 - \sin\phi'$ given by Broker-Ireland (1965) can be used. The direct shear test results were found to be very close to those given by Broker-Ireland (1965) For instance, the value of the friction angle obtained for normally consolidated clayey soils (specimen 1) using shear test and Broker-Ireland (1965) method were found to be 12° and $12^\circ.41$, respectively. This is highly satisfactory for engineering purposes.

For an oedemotor test, the Mohr-coulomb line of a normally consolidated soil can be drawn and the internal friction angle, ϕ , can also be found, This enables us to deduce the cohesion value, c , from $\tau = c' + \sigma_v \sin\phi$. In conclusion, the strength parameters related to the soil can be calculated.

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