# SYNTHESIS OF BASIC LINKAGES FOR INPUT-OUTPUT COORDINATION 

## MEHMET AKYURT*


#### Abstract

SUMMARY: Computer-aided graphical design procedures are introduced for the synthesis of basic linkages. The two-handled block concept is utilized to illustrate the design of four-bar linkages, slider-crank mechanisms, the first inversion of the slider-crank mechanism (FISC), as well their combinations for multi-position input-output coordination. It is shown that the coordination of input-output in the resulting mechanisms is accurate, and that the error is generally less than $3 \%$ of the range.


Key Words : Blocking, computer-aided, coordination, design, graphics, handle, linkage, mechanism, synthesis.

## INTRODUCTION

Virtually all machinery make use of some sort of a linkage in the production, delivery or application of power, motion, or function. A given linkage may be broken down into simpler sub linkages, or basic linkages, which when put together, form the linkage itself. The four-bar linkage and its inversions and the slider-crank mechanism and its inversions represent the most common of the basic linkages.

One of the applications of basic linkages arises when the motion of an output link is to be coordinated with that of an input link. Of particular interest is the multi-step coordination of four-bar linkages, slider-crank mechanisms, and FISC (first inversion of the slider-crank) mechanisms. Utilization of computer graphics permits the pragmatic synthesis of these linkages. Reasonably accurate designs are achieved after a few trials. The method itself is not complicated, and allows coordination at more than three positions.

In what follows we illustrate the procedures (1) by providing examples to the design of each of these basic linkages. A computer graphics package that utilizes the blocking concept is employed. The software must necessarily support two handles on the block.

## THE FOUR-BAR LINKAGE

A four-bar linkage similar to the one shown in Figure 1 used in some electric typewriters. Member $A_{o} A$ is

[^0]rotated clockwise via a cam (not shown) such that the type slug at C is moved rapidly against a platen roller (not shown) while $B B_{0} C$ rotates tawafwise (counter-clockwise). Let the required motion be specified as in Table 1, where $\alpha$ is a fixed angle to be determined by synthesis. Thus for given crank length $A_{o} A$ and coupler length $A B$, the design task is to ascertain the dimensions of $B_{0} B$ and $\alpha$, and the determination of the position vector $B_{o} A_{o}$ such that Table 1 is generated.


Figure 1: The four-bar linkage to be coordinated.

## Procedure

Starting from a suitable point $A_{0}$, draw the five radial positions of crank $A_{o} A$ (Figure 2a). Draw also five circles of radius $A B$, taking the tips of the rays of Figure $2 a$ as center. Note that the circles are drawn with different line types for ease in differentiation (Figure 2b).

Table 1: Data for the four-bar linkage.

| Inclination <br> of <br> $A_{0} A$ | Inclination <br> of $B_{o} C$ <br> (Required) | Inclination of $B_{o} C$ as <br> generated by <br> $A_{o} B B_{o} C$ of Figure 3 |
| :---: | :---: | :---: |
| 0 | 55 | 54.11 |
| 10 | 31 | 30.30 |
| 20 | 14 | 13.37 |
| 30 | 4 | 2.36 |
| 40 | 0 | -0.65 |



Figure 2: The procedure of synthesis for the four-bar linkage.

At a convenient location (Figure 2c) lay the five required radial positions of $B_{o} C$. Use any suitable scale for $B_{o} C$. Block the rays of Figure $2 c$, and set handles on the tips of rays numbered one and five. Set handle-1 at any point on the circumference of circle-1, and handle-2 on the circumference of circle-5, and move the block. Zoom if necessary. Check to see how closely the block fits. A good fit is obtained when the tip of each ray barely touches the circumference of the corresponding circle. Try setting the block at different locations until a satisfactory fit is obtained. A number of satisfactory solutions are generally possible. Figures $2 d$ and $2 e$ illustrate two such solutions.

The location of the vertex of the rays at the selected configuration is the location of fixed pivot $\mathrm{B}_{0}$. The length of ray-1 is the length of member $\mathrm{B}_{0} \mathrm{~B}$ and its inclination ( $139.42^{\circ}$ ) corresponds to that of $\mathrm{A}_{0} \mathrm{~A}$ at the starting position (Table 1). The position vector from $B_{o}$ to $A_{o}$ determines the length and inclination of the frame member. Figure 3 depicts the crossed four-bar linkage that results
from the block of $F$ igure $2 e$ for $A_{o} A=0.3$ and $A B=1$ units, where it is determined that $A_{0} B_{o}=0.736, B_{0} B=0.05$, and $A_{0} B_{0}$ makes an angle of $41.01^{\circ}$ with the horizontal. As $\mathrm{B}_{0} \mathrm{C}$ is to have an initial inclination of $55^{\circ}$ (Table 1), the fixed angle $\alpha$ must be $139.42-55=84.42^{\circ}$.


Figure 3: The resulting four-bar linkage.

Insertikon of the above data in the software package AL-YASEER (2-5) yields the output data for the mechanism of Figure 3 as listed in Table 1. It may be shown that transmission angles, within the range of interest, for the mechanism that results the block of Figure 2d are less favorable.

## THE SLIDER-CRANK MECHANISM

When the motion of the slider needs to be coordinated with the rotation of the crank in a slider-crank mechanism, the blocking technique can be utilized with ease. Consider, as an example, the garage-door guiding mechanism of Figure 4, where the door $A B$ is guided by $A_{o} A B$ from its nearly vertical (closed) position until it becomes nearly horizontal (open) while crank $A_{o} A$ is rotadeo tawafwise. Let it be known that $A B=2.2 \mathrm{~m}$, and that the variation of the $x$-location of the slider $B$ with respect to an origin at $A_{o}$ be given by Table 2. We wish to determine the crank length $A_{\circ} A$ and the eccentricity $I_{1}$ to generate this motion.

## Procedure

The displacements of the slider are plotted to an arbitrary scale (Figure 5a). With each slider location as center, circles of radius $A B=2.2 \mathrm{~m}$ are drawn (Figure 5 b ). The radial positions of the crank (Figure 5 c ) are next plotted at a suitable location and to an arbitrary scale. The rays of F igure 5 c are the blocked, with handles at the tips of ray-1 and ray-5. The block is set on the circles until
a satisfactory fit is obtained (Figure 5 d ). It is determined from Figure $5 d$ that $A_{o} A=1.034$, and $I_{1}=1.123 \mathrm{~m}$. The resulting mechanism is displayed in Figure 6, illustrating the five positions of the garage door. The locations of slider $B$ that are generated by the mechanism of $F$ igure 6 are listed in Table 2.


Figure 4: The garage door to be synthesized.

Table 2: Data for the garage door.

| Crank angle -0 | $\mathrm{x}_{\mathrm{B}}$ (required) | $\mathrm{x}_{\mathrm{B}}$ (generated) |
| :---: | :---: | :---: |
| 277 | -0.56 | -0.35 |
| 320 | -0.58 | -0.49 |
| 3 | -0.94 | -0.89 |
| 46 | -1.47 | -1.45 |
| 89 | -2.18 | -2.18 |



Figure 5: The procedure of synthesis for the slider-crank mechanism.


Figure 6: The resulting mechanism for the garage door, with the door at several locations.

## THE FISC MECHANISM

Synthesis of FISC mechanisms by the aid of computer graphics is also straightforward. Consider, as an example, the wiper mechanism of Figure 7. Let it be given that crank $A_{o} A$ has a length of 5 units, and that the required variation of $T_{9}$ with $T_{7}$ is to be as listed in Table 3. We wish to determine the frame vector $B_{o} A_{o}$ as well as length $B_{o} B$ of the output member.


Figure 7: The wiper mechanism

## Procedure

Figure 8 outlines the design procedure that consists of plotting, to a suitable scale, the seven radial positions of crank $\mathrm{A}_{0} \mathrm{~A}_{o}$ (Figure 8a) and of the output member $\mathrm{B}_{0} \mathrm{~B}$ (Figure 8 b ). A handle is now placed at the vertex of the rays of Figure 8 b , which is laid out to an arbitrary scale, and another on ray-1 at a distance of about a third the ray length from the vertex. Set two points, one at an arbitrary location, and the second point at the outer tip of ray-1 of

Table 3: Data for the wiper mechanism.

| $\mathrm{T}_{7} \mathbf{- 0}^{0}$ | $\mathrm{~T}_{9}$ (required) | $\mathrm{T}_{9}$ (generated)+1.70 |
| :---: | :---: | :---: |
| -51 | 66.24 | 66.24 |
| -21 | 61.69 | 62.55 |
| 9 | 67.72 | 68.52 |
| 39 | 77.24 | 77.86 |
| 69 | 88.16 | 88.56 |
| 99 | 99.62 | 99.78 |
| 129 | 111.08 | 110.98 |

Figure 8a. The object is to have each ray of the block to barely touch the tip of the corresponding ray of F igure 8 a . Move the block from location to location, zooming to increase accuracy, until a satisfactory fit is achieved. Figure 8c illustrates one such solution where it is determined that $\mathrm{A}_{0} \mathrm{~B}_{0}=8.286$ units and making an angle of $97.53^{\circ}$ with the horizontal. The length of $B_{0} B_{o}$ is determined to be $\geq 13.285$ units. Table 3 lists the generated output angles for this mechanism. Incorporated in the results is an addition of $1.7^{\circ}$ due to the difference of orientation of ray-1 of the block of Figure 8c from that of Figure 8 b . Figure 9 shows the resulting mechanism.


Figure 8: The procedure of synthesis for the wiper mechanism.

## BASIC MECHANISMS IN TANDEM

The process of synthesis of mechanisms that involve more than one basic linkage proceeds essentials as outlined above for individual basic linkages. To illustrate, consider the hacksaw mechanism of Figure 10. We observe that the mechanism consists of three basic linkages, i.e., FISCs $\mathrm{A}_{o} \mathrm{AB}_{0} \mathrm{~B}$ and $\mathrm{B}_{0} \mathrm{BC}{ }_{o} \mathrm{C}$, and the slidercrank mechanism $\mathrm{C}_{0} \mathrm{CD}$. Let it be required to design this mechanism to achieve the coordination listed in Table 4, where input is the angle of crank $\mathrm{A}_{0} \mathrm{~A}$, and output refers to the displacements, relative to point $\mathrm{C}_{0}$, of the blade at D .


Figure 9: The resulting mechanism for the wiper mechanism.


Figure 10: The hack-saw mechanism.

Table 4: Design data for the hack-saw mechanism.

| Input- 0 | Output (required) <br> Units | Output (generated) <br> Units |
| :---: | :---: | :---: |
| -80 | -8.42 | -8.59 |
| -50 | -6.79 | -7.11 |
| -20 | -9.74 | -9.81 |
| 10 | -14.98 | -14.78 |
| 40 | -16.28 | -16.04 |
| 70 | -16.18 | -15.94 |

## Procedure

Starting with input crank $A_{o} A$, additional design information may be provided, or rational estimates may be made by the designer himself. Thus let the length of $A_{o} A$ be 3 units, and the input and the output for FISC $A_{0} A B{ }_{o} B$ (Figures 10 and 11) be given by Table 5. Figure 12 outlines the corresponding procedure. It is found thus that $\mathrm{A}_{0} \mathrm{~B}_{0}=8.422, \mathrm{~B}_{0} \mathrm{~B}>11.422$ units, and $\mathrm{T}_{6}=197.33^{\circ}$. We let $B_{o} B=12$ units. The output generated by this mechanism is also listed in Table 5.


Figure 11: The first FISC of the hack-saw.

Table 5: Data for the FISC $\mathrm{A}_{0} \mathrm{AB}_{0} \mathrm{~B}$.

| $\mathrm{T}_{7}-0$ | $\mathrm{~T}_{9}$ (required)- 0 | $\mathrm{~T}_{9}$ (generated)- 0 |
| :---: | :---: | :---: |
| -80 | 216.45 | 216.45 |
| -50 | 218.75 | 218.64 |
| -20 | 214.72 | 214.55 |
| 10 | 201.76 | 201.80 |
| 40 | 185.78 | 186.2 |
| 70 | 177.31 | 177.92 |



Figure 12: The procedure of synthesis for the first FISC.

Taking up the FISC $\mathrm{B}_{0} \mathrm{BC}_{0} \mathrm{C}$ next, where $\mathrm{B}{ }_{0} \mathrm{~B}$ is now the crank, let it be given that the relationship of $\mathrm{T}_{9}$ and $\mathrm{T}_{2}$ (Figure 13) is given by Table 6. Figure 14 presents the design procedure whereby it is determined that $C_{0} B_{0}=14.78, C_{0} C \geq 10.13$ units, and $T_{6}=40.53$.

The synthesis of slider crank mechanism $\mathrm{C}_{0} \mathrm{CD}$ (Figure 15) depends on the data provided by Tables 1 and 6 , as outlined in Table 7. Letting the length of the connecting rod CD be equal to 15 units, and following the design procedure outlined in Figure 16 we find that $I_{1}=9.183$ and $C_{0} C=12.317$ units.


Figure 13: The secondFISC of the hack-saw.

Table 6: Data for the FISC $\mathrm{B}_{o} \mathrm{BC}_{o} \mathrm{C}$.

| $\mathrm{T}_{9}-0$ | $\mathrm{~T}_{2}$ (required)- 0 | $\mathrm{~T}_{2}$ (generated)- 0 |
| :---: | :---: | :---: |
| 216.45 | 58.67 | 59.00 |
| 218.64 | 48.63 | 50.18 |
| 214.55 | 66.28 | 65.82 |
| 201.80 | 91.73 | 90.55 |
| 186.22 | 96.64 | 96.36 |
| 177.92 | 95.83 | 95.89 |



Figure 14: The procedure of synthesis for the second FISC.

The mechanism that resulted from the current synthesis is shown in Figure 17. The output generated by this eight-bar linkage is summarized in Table 7.


Figure 15: The slider-crank mechanism of the hack-saw.

Table 7: Data for slider-crank mechanism $C_{0} C D$.

| $\mathrm{T}_{7}$ | $\mathrm{~T}_{2}$ | $\mathrm{x}_{\mathrm{D}}$ (required) | $\mathrm{x}_{\mathrm{D}}$ (generated) |
| :---: | :---: | :---: | :---: |
| -80 | 59.00 | -8.42 | -8.59 |
| -50 | 50.18 | -6.79 | -7.11 |
| -20 | 65.82 | -9.74 | -9.81 |
| 10 | 90.55 | -14.98 | -14.78 |
| 40 | 96.36 | -16.28 | -16.08 |
| 70 | 95.89 | -16.18 | -15.95 |



Figure 16: The procedure of synthesis for slider-crank mechanism of the hack-saw.


Figure 17: The resulting hack-saw.

## CONCLUSIONS

The two-handled blocking method introduced here possesses the potential to become a powerful technique for the computer-aided graphical design of mechanisms. The method allows the rapid determination of linkage dimensions. Powerful tools of modern design packages such as zoom and intersect are effectively used to achieve highly accurate designs. One outstanding feature of the approach is that the designer is able to synthesize for input-output coordination without losing sight of the physical situation. The results are relatively accurate, the resulting coordination errors beign generally below 3\% of the range.

## REFERENCES

1. Akyurt M : Computer-Aided Design of Mechanisms, Mechanical Engineering Dept, King Abdulaziz University, Jeddah, p 203, 1991.
2. Mannaa AR, Mansour MA, Akyurt M : Mechanism analysis by the aid of AL-YASEER, Proc Tenth National Computer Conf, Feb 28-March 3, KAU, Jeddah, Sci Publ Center, KAU, Jeddah, 2:857-870, 1988.
3. Mannaa AR, Dahlawi F, Akyurt M : Kinematic analysis via virtual links, Computers in Industry, 10:261-266, 1988.
4. Akyurt M : Analysis of Mechanisms and Machinery, Center for Scientific Publications, King Abdulaziz University, J eddah, p 487 (accepted).
5. Akyurt $M$ : On the use of computers during the undergraduate teaching of mechanisms and machine dynamics (in Turkish), J Mech Design Prod (METU-Ankara), 2:92-98, 1991.

Correspondence:
Mehmet AKYURT
Department of Mechanical E ngineering, King Abdullaziz University, J eddah 21413, SAUDI ARABIA.


[^0]:    * From Department of Mechanical Engineering, King Abdullaziz University, J eddah 21413, S audi Arabia.

