Comparison of metaheuristics on multi objective (cost&CO₂) optimization of RC cantilever retaining walls

Betonarme konsol istinat duvarlarının çok amaçlı(maliyet ve karbondioksit) optimizasyonunda meta-sezgisel yöntemlerin karşılaştırılması

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Abstract

In this study, performance of meta-heuristic methods on optimum design of reinforced concrete (RC) retaining wall has been investigated. The maximization of the cost, the CO₂ emission and multi-objective of the cost+CO₂ functions are described as objective functions of the optimization problem. There are thirteen design variables are defined in the optimization problem. Eight of these variables are the cross sectional dimensions of the retaining wall. The other five design variables are the reinforcement detailing of wall members. Flexural and shear strength requirements, minimum and maximum cross section areas of the reinforcement bar, the requirement length for reinforcement details and the factor of safety for failure modes are defined as constraints functions of the optimization problem. The flexural and shear strength requirements, minimum and maximum limitations of the reinforcement bar areas are adopted from American Concrete Institute (ACI 318-14) design code. In order to test performance of the presented optimization methods literature design examples are used. In addition, efficiency of steel and concrete classes on optimum CO₂ emission and cost have been investigated by using different steel and concrete classes.

Keywords: Biogeography, Social spider, Optimization, Reinforced concrete retaining walls, Sustainable design

1 Introduction

RC cantilever retaining wall is one of the favorite type of retaining structure. Although concrete material seems to be less expensive than steel, it causes more CO₂ emissions to environment and also global warming. Thus, minimizing CO₂ emissions and cost should be considered in the optimum design of the RWs. However, reaching these objectives is difficult as discrete design variables and nonlinear functions are included in the optimization problem. Stochastic search techniques are great tools for the solution of the optimization problem. Ant colony optimization, hunting, particle swarm, firefly and bat algorithms are popular stochastic search techniques that have been mostly used in structural optimization problems for 10 years.

The BBO method [1] is the recent stochastic search technique which mimics the theory of island biogeography. The theory consist two main behaviors. These are speciation (the determine performance of new animals), the extinction of animals and the migration of animals between islands. Despite being the recent optimization algorithm, the BBO has been used in many optimization problems such as: economic dispatch solution [2], power flow problem [3], cognitive radio systems [4], security audit trail analysis [5], satellite remote sensing images [6], AC transmission system devices [7], approach for segmentation of human head [8], profit maximization of a generation company [9], flexible job shop scheduling problem [10], mirrored traveling tournament problem [11]. However, there is no major study about application of BBO for optimum design of RC structures. Hence, this study is an original study as it includes application of BBO in the optimizations of the RC structures.

A new stochastic search algorithm and an innovative approach called Social Spider Optimization (SSO) technique has been developed in 2013 by adopting movement and mating behaviors of spider colony [12]. Despite being the new technique, the SSO algorithm has been applied on many fields such as: dispatch of thermal power unit [13], design of plug-in electric vehicle [14], wind tribune systems [15], feed forward neural networks learning [16], optical flow methods parameters [17], field weakening control of a DC motor [18] and energy theft detection systems[19]. However, any article
about an application of the SSO algorithm for RW design problems has been found in the literature. Hence, this study is the first study which uses the SSO algorithm to the RW design.

There are numerous studies on optimum design of RC retaining walls: [20]-[27]. However, few researches [28] took into account the environmental effects of retaining wall design. In structural engineering, the environmental aspects have been considered in the recent years [28]-[36]. It is concluded from these studies that RC structure designs having economically low-CO₂ emission can be obtained even from complex design problems.

Outline of the remainder of the study is described as follows. Section 2 briefly describes mathematical modeling of the optimization problem. The BBO and SSO algorithms for the RC Cantilever Retaining Walls are described in the Sections 3 and 4. Parametric study of the optimization algorithm is given in the Section 5. Details of the design examples and their results are given in Section 6. Conclusions of the study are provided in Section 7.

2 Mathematical model

Optimum design of RC retaining wall problems are defined as the selection of dimensions of retaining wall, number and diameter of reinforcements such that safety, stability and stress limitations specified by the concrete building code are satisfied. It is also necessary to consider the economic and environmental aspects in this selection. In this study, three objective functions are defined. The first is the minimization of the cost of the retaining wall which is expressed in Equation (1).

\[ f_{\text{cost}}(X) = C_cW_c + C_vV_c \]  

(1)

Where, \(X\) is the vector which contains the sequence numbers of design variables, \(C_c\) is the unit cost of concrete, \(W_c\) is the weight of steel per unit length of the wall, and \(V_c\) is the volume of concrete per unit length of the wall.

The second objective function is minimization of the CO₂ emission of the retaining wall which is expressed in Equation (2).

\[ f_{\text{CO}_2}(X) = \sum_{i=1}^{N_{\text{material}}} A_i \rho_i E\text{CO}_2 \]  

(2)

Where \(A\), \(E\text{CO}_2\), and \(\rho\) are the cross sectional area, CO₂ emission and density of the structural materials respectively. \(N_{\text{material}}\) is the number of materials defined in the structure design problem. CO₂ emissions of the structural materials are adopted from literature studies [33],[36] which are shown in Table 1.

In the study, weighted aggregate of the cost and the CO₂ functions of the RC retaining walls are also considered as the objective function. Mathematical formulation of the objective function is described in Equation (3).

\[ f_{\text{aggr}} = \zeta_{\text{cost}}f_{\text{cost}} + \zeta_{\text{CO}_2}f_{\text{CO}_2} \]  

(3)

Where \(\zeta_{\text{cost}}\) and \(\zeta_{\text{CO}_2}\) are non-negative weights which are taken as 1 in this study [31].

Thirteen design variables are defined in this mathematical model. Eight of these variables describe geometry of the RC cantilever retaining wall and the other five design variables are the reinforcement detailing of wall members (see Figures 1 and 2). Upper and lower limits of cross section dimensions of the retaining wall are illustrated in Table 2.

Reinforcement design variables are considered as discrete design variables which are defined as \(n_d\) (n: number of bars, d: diameter of bars): the number and diameter of the first stem reinforcement which extends to toe (R1), the number and diameter of the second stem reinforcement which extends to toe (R2), the number and diameter of additional toe reinforcement together (R3), the number and diameter of heel reinforcement together (R4), and the number and diameter of key reinforcement together (R5)

Table 1: Unit CO₂ emissions and unit price of the structural materials.

<table>
<thead>
<tr>
<th>Material</th>
<th>Strength</th>
<th>Unit Price</th>
<th>CO₂ Emission</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>24 MPa</td>
<td>59.76 $/m³</td>
<td>304.75 CO₂/m³</td>
</tr>
<tr>
<td></td>
<td>27 MPa</td>
<td>62.50 $/m³</td>
<td>324.76 CO₂/m³</td>
</tr>
<tr>
<td></td>
<td>30 MPa</td>
<td>65.65 $/m³</td>
<td>344.54 CO₂/m³</td>
</tr>
<tr>
<td>Steel</td>
<td>400 MPa</td>
<td>0.742 $/kg</td>
<td>0.3857 CO₂/kg</td>
</tr>
<tr>
<td></td>
<td>500 MPa</td>
<td>0.770 $/kg</td>
<td>0.3962 CO₂/kg</td>
</tr>
</tbody>
</table>

Table 2: Lower and upper limits of cross sectional design variables [20],[24],[39],[40].

<table>
<thead>
<tr>
<th>Design Variables</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>X1</td>
<td>0.40H</td>
<td>0.80H</td>
</tr>
<tr>
<td>X2</td>
<td>0.10H</td>
<td>0.60H</td>
</tr>
<tr>
<td>X3</td>
<td>0.20 m</td>
<td>0.50 m</td>
</tr>
<tr>
<td>X4</td>
<td>0.20 m</td>
<td>0.40 m</td>
</tr>
<tr>
<td>X5</td>
<td>0.20 m</td>
<td>0.3H</td>
</tr>
<tr>
<td>X6</td>
<td>0.5H</td>
<td>0.8H</td>
</tr>
<tr>
<td>X7</td>
<td>0.20 m</td>
<td>0.40 m</td>
</tr>
<tr>
<td>X8</td>
<td>0.20 m</td>
<td>0.90 m</td>
</tr>
</tbody>
</table>

Figure 1: Design variables of the retaining wall.

There are thirty one constraint functions defined in the optimization. First three of them can be grouped into stability constraint functions which are described as: overturning, sliding and bearing capacity constraint functions in (4)-(6). The fourth constraint function is defined from the "no tension" condition (see Equation (7)).
Figure 2: Reinforcement description.

Table 3: Reinforcement design variables.

<table>
<thead>
<tr>
<th>#</th>
<th>Value</th>
<th>#</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>No Bar</td>
<td>34</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>φ</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>φ</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>φ</td>
<td>4</td>
</tr>
<tr>
<td>13</td>
<td>2</td>
<td>φ</td>
<td>0</td>
</tr>
<tr>
<td>14</td>
<td>2</td>
<td>φ</td>
<td>2</td>
</tr>
<tr>
<td>23</td>
<td>.</td>
<td>109</td>
<td>10</td>
</tr>
<tr>
<td>24</td>
<td>3</td>
<td>φ</td>
<td>0</td>
</tr>
</tbody>
</table>

\[ g_1(X) = \frac{FS_{overturning}}{FS_{provided \ for \ over}} - 1 \geq 0 \quad (4) \]

\[ g_2(X) = \frac{FS_{sliding}}{FS_{provided \ for \ side}} - 1 \geq 0 \quad (5) \]

\[ g_3(X) = \frac{FS_{bearing}}{FS_{provided \ for \ bear}} - 1 \geq 0 \quad (6) \]

\[ g_4(X) = q_{\text{min}} \geq 0 \quad (7) \]

The 5th-14th constraint functions, defined in the optimization problem, are capacity constraint functions which are formulated in Equation (8) and Equation (9),

\[ g_{5-9}(X) = \frac{M_u}{M_u^{\text{Critical section}}} - 1 \leq 0 \quad (8) \]

\[ g_{10-14}(X) = \frac{V_u}{V_u^{\text{Critical section}}} - 1 \leq 0 \quad (9) \]

where \( M_u \) is the ultimate resistance moment, \( M_d \) is the design moment, \( V_u \) is the ultimate shear capacity, \( V_d \) is the design shear force and \( \text{Critical section} \) refers to either the stem, toe, heel, and key of the retaining wall (see Figure 1). Reinforcement arrangement constraint functions are also considered in this optimization problem which is described as:

\[ g_{15-19}(X) = \frac{A_{\text{min}}^{\text{section}}}{A_s} - 1 \leq 0 \quad (10) \]

where \( A_s^{\text{min}} \) and \( A_s^{\text{max}} \) are the minimum and maximum reinforcement areas defined in code [37]. The last five constraint function groups are the geometric limitation functions of size, reinforcement bars and clear cover limitation of the retaining wall which are formulated in (11)-(16).

\[ g_{25}(X) = \frac{X_2 + X_3}{X_1} - 1 \leq 0 \quad (11) \]

\[ g_{26}(X) = \frac{X_2 + X_3}{X_1} - 1 \leq 0 \quad (12) \]

\[ g_{27}(X) = \frac{X_5}{X_1} - (2, c) - 1 \leq 0 \quad (13) \]

or

\[ g_{28}(X) = \frac{L_d}{X_1 - X_2 - c} - 1 \leq 0 \quad (14) \]

and

\[ g_{29}(X) = \frac{12d_b}{X_5 - c} - 1 \leq 0 \quad (15) \]

\[ g_{31}(X) = \frac{S_{\text{net}}}{S_{\text{max}}^{\text{section}} - 1 \geq 0 \quad (16) \]

where \( L_d \) is the minimum development length; \( L_d \) is the minimum hook development length; \( d_b \) is the diameter of the hooked bar; \( S_{\text{net}} \) is the clear spacing, and \( S_{\text{max}} \) is the maximum clear spacing (See Figure 3).

3 Bio-geography based optimization algorithm

BBO algorithm is firstly introduced by D. Simon in 2008 [1], [39] by adopting the theory of island biogeography. The BBO algorithm describes the extinction and migrations of species between islands. The island is defined as an isolated area for species. Two main indexes are related to the extinction and migrations of species between islands. These are habitat suitability index (HIS) and suitability index variables (SIV). HIS parameter describes life quality of habitats in Islands. SIV index characterize habitability which can be considered as independent variables of the habitat. If the habitats have high HIS index, the islands provide good life standards to the species and a large number of them live in the habitats. These habitats have a low species immigration rate because they are already nearly saturated. This assumption is used in the BBO algorithm for carrying out migration. Relationship between number of species and rate of emigration and immigration is illustrated in Figure 4 [1].

In the Figure 4, \( \lambda \) is the immigration rate; \( \mu \) is the emigration rate; \( I \) is the maximum immigration rate; \( E \) is the maximum emigration rate; \( S_o \) is the equilibrium number of species and \( S_{\text{max}} \) is the maximum number of species.

Figure 3: Reinforcement details.
In the BBO algorithm, the new candidate design is generated by modifying the independent design variable of the old design which is related to the immigration rate of the design variable. If an independent variable is to be modified, then the emigrating candidate design is chosen by using the roulette wheel selection method that is related to the immigration probability (see Equation (17)).

\[
P(x_j) = \frac{\mu_j}{\sum_i=1^{N} \mu_i}, \quad j = 1, ..., N
\]  

(17)

Where \( N \) is the size of population. One more factor is related to the generation of the new design called Mutation. This factor is used to increase the number of species in the islands. Mutation probability of each design is described as follows:

\[
m(s) = m_{\text{max}} \left( 1 - \frac{P}{P_{\text{max}}} \right)
\]  

(18)

Where \( m_{\text{max}} \) maximum mutation probability defined by user \( P \), probability of species and \( P_{\text{max}} \) is the maximum probability species.

The optimization method based on BBO algorithm tries to find the optimum geometry and reinforcement details of the RC Cantilever Retaining Walls. The steps of the optimization method are described as follows:

**Step1:** In first step, number of population size \( (N) \) RC cantilever retaining wall designs are generated randomly using Equation (19). Then, the designs are evaluated and penalized values of their objective functions are calculated using Equation (20).

\[
x_i = x_{li} + r(x_{ui} - x_{li}), \quad i = 1, ..., N
\]  

(19)

\[
f_p = f \cdot (1 + C)\varepsilon
\]  

(20)

Where \( f \) is the value of the objective functions described in Equations (1-3), \( C \) is the summation of constraint violations calculated using constraint functions stated by Equations (4-16), \( \varepsilon \) is penalty coefficient which is taken as 2 and \( r \) is a random number between \((0, 1)\). In general form, constraint violations are calculated as:

\[
C_i = \begin{cases} 
0 & g_i(X) \leq 0 \\
g_i(X) & g_i(X) > 0 
\end{cases}, \quad i = 1, 2, ..., NC
\]  

(21)

Where, \( g_i(x) \) is the \( i^{th} \) constraint function and \( NC \) is the number of constraint functions defined in the optimum design problem.

**Step2:** Firstly the elite designs which have the lowest penalized objective function values are determined in this step. Then, immigration and emigration rates of the designs are calculated as follows:

\[
\mu_j = \frac{N + 1 - j}{N + 1}, \quad \lambda_j = 1 - \mu_j, \quad j = 1, 2, ..., N
\]  

(22)

**Step3:** The immigration and the emigration parts are performed in the step. In the immigration part, the RC cantilever retaining wall designs are updated with respect to their immigration rates by modifying the independent design variables. A change criterion for the independent design variable is described as:

\[
r < \lambda_j^k, \quad j = 1, 2, ..., N, \quad k = 1, 2, ..., NDV
\]  

(23)

Where \( NDV \) is number of design variables defined in the optimization problem. In emigration part, the designs are modified by using the roulette wheel selection method that is related to the emigration probability described in the Equation (17).

**Step4:** The designs are mutated in this step. Mutation probabilities of the designs are calculated using Equation (18). If mutation is performed, new RC cantilever retaining wall design is generated randomly. The steps 2 to 4 are repeated until a pre-assigned maximum number of iterations are completed.

### 4 Social spider optimization algorithm

Social spider optimization (SSO) algorithm is one of the newest meta-heuristic search algorithm mimics the behaviors of a spider colony. In the spider colony, male and female spiders perform different tasks called movement and mating. In the movement stage, each spider moves to new position which is related to vibrations of its and other colony members. The vibrations of the spiders depend on the gender, distance between the spiders and their weights. In the mating stage, the each male spider having higher weight (dominant males) finds the suitable female spiders in its range and generates a new spider.

The main steps of the SSO algorithm for the optimum design of the optimization problem are described as follows;

**Step 1:** Initial parameters of the SSO algorithm, which are the number of female spiders \((N_f)\) and the number of male spiders \((N_m)\), are determined in this step using equations (24) and (25) respectively.

\[
N_f = \text{round}((0.9 - 0.25 \cdot r)N_3)
\]  

(24)

\[
N_m = N_k - N_f
\]  

(25)

Where, \( \text{round} \) is a function which rounds to the value of the nearest integer.

**Step 2:** Initial retaining wall designs, assigned to the female \( \left(f_i \right) \) and the male \( \left(m_{ij} \right) \) spiders, are generated randomly using equations (26) and (27). Then, penalized objective function values of the designs \( \left(f_i \right) \) are calculated using equation (20).

\[
f_{ij} = x_{ui} + r(x_{ui} - x_{li}), \quad i = 1, 2, ..., N_f, \quad j = 1, 2, ..., NDV
\]  

(26)

\[
m_{kj} = x_{ui} + r(x_{ui} - x_{li}), \quad k = 1, 2, ..., N_m, \quad j = 1, 2, ..., NDV
\]  

(27)

**Step 3:** After the evaluation process, the spider having the lowest objective function value and called the best spider \( S_0 \) and the spider having the highest objective function value and
called the worst spider \( S_{\text{w}} \) are determined. Then, the weights of the spiders are calculated as follows:

\[
w_t = \frac{f_{\text{high}} - f_t}{f_{\text{high}} - f_{\text{low}}} \quad i = 1, \ldots, N_f
\]

(28)

Where, \( f_{\text{high}}, f_{\text{low}} \) and \( f_t \) are objective function values of the worst spider, the best spider and the \( t^{th} \) spider respectively.

**Step 4:** In this step, all spiders move to new positions (generate new designs). In the colony, the female and the male spiders use different movement strategies given as follows:

\[
f_{k+1} = \left\{ \begin{array}{ll}
f_t^k + \alpha \cdot \text{vibc}((x_{i,j}^{f_k})_n + \beta \cdot \text{rand}(r \cdot 0.5) - \text{PF}) & \text{if } w_{m,n} > w_m \\
+f_b^k + \alpha \cdot \text{vibb}((x_{i,j}^{f_k})_n + \beta \cdot \text{rand}(r \cdot 0.5) - 1.0 - \text{PF}) & \text{if } w_{m,n} \leq w_m \\
\end{array} \right.
\]

(29)

\[
m_{k+1} = \left\{ \begin{array}{ll}
m_{i,j}^k + \alpha \cdot \text{vibb}(x_{i,j}^{m_k}) + \beta \cdot \text{rand}(r \cdot 0.5) & \text{if } w_{m,n} > w_m \\
+m_{i,j}^k + \alpha \cdot \text{vibb}(x_{i,j}^{m_k}) - \beta \cdot \text{rand}(r \cdot 0.5) & \text{if } w_{m,n} \leq w_m \\
\end{array} \right.
\]

(30)

Where, \( \alpha, \beta \) and \( \beta \cdot \text{rand} \) are the random numbers between \( (0, 1) \); \( x_{i,j}^{f_k} \) and \( x_{i,j}^{m_k} \) are the \( t^{th} \) design variable of the nearest and the best spider; \( \text{vibc} \) is the vibration between the \( i^{th} \) spider and the nearest spider to the \( m^{th} \) spider calculated using equation (31); \( \text{vibb} \) is the vibration between the \( i^{th} \) spider and the best spider calculated using equation (32); \( \text{vibf} \) is the vibration between the \( i^{th} \) spider and the nearest female spider to the \( j^{th} \) spider calculated using equation (33); \( \text{wmed} \) is the weight of the median spider; \( k \) is the iteration number; \( PF \) is the female movement parameter between \( (0, 1) \).

\[
\text{vibc} = 0 \quad \text{if } W_j \geq W_o
\]

(31)

\[
\text{vibc} = w_p \cdot \sum_{i=1}^{n}(x_{i,j} - x_{o,j})^2 \quad \text{if } W_j < W_o
\]

(32)

\[
\text{vibf} = w_p \cdot \sum_{i=1}^{n}(x_{i,j} - x_{o,j})^2 
\]

(33)

Where, \( x_{i,j}^{f_k} \) is the \( j^{th} \) design variable of the nearest female spider; \( w_o \) and \( w_f \) are the weights of the nearest spider, the best spider and the nearest female spider respectively. After the movement, the new designs are evaluated, their penalized costs are calculated using equation (20) and the colony is updated.

**Step 5:** In this step, the mating is performed by the dominant male spiders and the female spiders within the range of the dominant spiders. The dominant male spiders are determined by selecting male spiders whose weights are heavier than weight of the median spider. The female spiders in the range of the dominant male spiders are determined using following conditions:

\[
\begin{array}{l}
\sum_{i=1}^{n}(x_{i,j} - x_{i,j}^{f_k})^2 \leq \frac{\sum_{i=1}^{N_{\text{med}}}(x_{i,j} - x_{i,j})^2}{2 \cdot \sqrt{NDV}} \\
\end{array}
\]

(34)

Where, \( x_{i,j}^{f_k} \) is the \( j^{th} \) design variable of the \( m^{th} \) dominant spider; \( x_{i,j}^{f_k} \) is the \( j^{th} \) design variable of the female spider; and \( N_{\text{med}} \) is the number of dominant male spiders. If there are no female spiders in the range of the dominant male spiders, mating operation is not performed for the dominant male spider. After determination of female spiders, the new design is generated.

Then, the new design is evaluated, its penalized cost is calculated using equation (20). If cost of the new design is less than the worst design in the colony, the worst design is replaced with the new design and the colony is updated.

**Step 6:** The termination criteria, which is the reaching maximum iteration number, is checked. If the termination criteria are satisfied, the algorithm is stopped. Otherwise, steps 3 to 6 are repeated.

### 5 Parametric study of the optimization algorithms

The selection of the search parameters values is considerably vital on the performance of the optimization algorithms. Thus, parametric study is demanded to find suitable values of the search parameters. The cost optimization the 3.5 m height cantilever RW is selected for the parametric study. Detail of the structure is described in section 6.1. The RW is optimized by using the optimization algorithms with different values of the search parameters: \( N=25, 50 \) and 100, Mutation probability=0.1%, 0.5% 1% and 5% and number of elite population=0.04*N, 0.1*N for the BBO algorithm; \( N=25, 50 \) and 100, PF=0.3, 0.4, 0.5, 0.6 0.7 and 0.8 for the SSO algorithm. In each test, the example is optimized 50 times using different seed values. The average and best optimum costs obtained from these tests are illustrated in Tables 4 and 5. According to the tables, the most convenient search parameters arebolded in the tables.

<table>
<thead>
<tr>
<th>Design Height</th>
<th>Mean Cost</th>
<th>Best Cost</th>
<th>Mean Cost</th>
<th>Best Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.5 m</td>
<td>122.1</td>
<td>120.7</td>
<td>118.1</td>
<td>116.3</td>
</tr>
<tr>
<td>5.2 m</td>
<td>121.6</td>
<td>120.3</td>
<td>118.8</td>
<td>116.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Design Height</th>
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<td>5.2 m</td>
<td>121.6</td>
<td>120.3</td>
<td>118.8</td>
<td>116.5</td>
</tr>
</tbody>
</table>

### 6 Design examples

#### 6.1 Case 1: Comparison of metaheuristics on cost optimization of retaining walls

In this case, two design examples (3.5 m and 5.2 m Height Retaining Walls) are solved using presented algorithms which
are previously used in literature [24],[39],[40]. Only cost optimizations are performed in these design examples and obtained results are compared to literature results (Harmony Search (HS) [24], Classical Firefly Algorithm (CFFA) and Adaptive Firefly Algorithm (AFFA) [39]). The input data of the examples are shown in Table 6. The search parameters of the literature algorithms are illustrated in Table 7.

Table 6: Search parameters of literature algorithms.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Parameter</th>
<th>Ex.1</th>
<th>Ex.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>HS</td>
<td>HMSC</td>
<td>0.9</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>PAR</td>
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<td>0.4</td>
</tr>
<tr>
<td></td>
<td>NFF</td>
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<td>100</td>
</tr>
<tr>
<td></td>
<td>B0</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>CFFA</td>
<td>Acoeff</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>RP</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>NFF</td>
<td>80</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>B0</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>AFFA</td>
<td>Acoeff</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>RPmin</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>RPmax</td>
<td>0.8</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Shrinkage and temporary reinforcement area are computed as 0.002% of the cross-sectional area of the retaining wall and the length of these bars is taken as 100 cm (per meter). The design examples are optimized 50 times using different seed values. After 50 runs, the average costs and corresponding standard deviations on optimized costs are illustrated in Table 8. Design variables and cost details of the best optimum design and literature results are illustrated in Tables 9-11. The search histories of the best optimum designs of each algorithm are shown in Figures 5 and 6.

In First example, the minimum cost is obtained as $113.67 by utilizing the BBO algorithm. This value is 3.799% less than the cost of HS’s optimum design, 0.699% less than the cost of AFFA’s optimum design, 0.822% less than the cost of SSO’s optimum design and 15.11% less than cost of CFFA’s optimum design. In the second example, the best design obtained using the SSO algorithm ($171.89). It is also remarked that all obtained designs satisfy design limitations described in section 2.

Table 7: Statistical results of examples.

<table>
<thead>
<tr>
<th></th>
<th>Best</th>
<th>Average</th>
<th>St. Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ex.1</td>
<td>BBO</td>
<td>113.93</td>
<td>118.70</td>
</tr>
<tr>
<td></td>
<td>SSO</td>
<td>114.6</td>
<td>117.00</td>
</tr>
<tr>
<td>Ex.2</td>
<td>BBO</td>
<td>180.16</td>
<td>198.40</td>
</tr>
<tr>
<td></td>
<td>SSO</td>
<td>171.89</td>
<td>182.16</td>
</tr>
</tbody>
</table>

Table 8: Cost details of the optimum designs for the ex. 1.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Vol.Conc. (m³)</td>
<td>2.03</td>
<td>1.99</td>
<td>1.94</td>
<td>2.17</td>
<td>1.93</td>
</tr>
<tr>
<td>Weight tot. (kg)</td>
<td>92.20</td>
<td>95.21</td>
<td>77.47</td>
<td>110.05</td>
<td>93.24</td>
</tr>
<tr>
<td>Cost Conc. ($)</td>
<td>81.10</td>
<td>75.58</td>
<td>92.82</td>
<td>86.78</td>
<td>77.17</td>
</tr>
<tr>
<td>Cost Ex. ($)</td>
<td>36.80</td>
<td>39.09</td>
<td>37.13</td>
<td>44.02</td>
<td>37.29</td>
</tr>
<tr>
<td>Cost Total ($)</td>
<td>117.90</td>
<td>113.67</td>
<td>114.60</td>
<td>130.80</td>
<td>114.46</td>
</tr>
</tbody>
</table>

Table 9: Optimum values of design variables for the ex.1.

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>X1</td>
<td>2.70</td>
<td>2.61</td>
<td>2.63</td>
<td>3.26</td>
<td>2.91</td>
</tr>
<tr>
<td>X2</td>
<td>1.60</td>
<td>1.42</td>
<td>1.48</td>
<td>1.47</td>
<td>1.46</td>
</tr>
<tr>
<td>X3</td>
<td>0.35</td>
<td>0.32</td>
<td>0.34</td>
<td>0.46</td>
<td>0.46</td>
</tr>
<tr>
<td>X4</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>X5</td>
<td>0.35</td>
<td>0.31</td>
<td>0.32</td>
<td>0.35</td>
<td>0.34</td>
</tr>
<tr>
<td>X6</td>
<td>1.90</td>
<td>2.40</td>
<td>2.40</td>
<td>2.80</td>
<td>2.61</td>
</tr>
<tr>
<td>X7</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>X8</td>
<td>0.60</td>
<td>0.84</td>
<td>0.76</td>
<td>0.20</td>
<td>0.64</td>
</tr>
<tr>
<td>R1</td>
<td>10.610</td>
<td>10.610</td>
<td>10.610</td>
<td>2.926</td>
<td>5.416</td>
</tr>
<tr>
<td>R2</td>
<td>10.94</td>
<td>9.412</td>
<td>8.410</td>
<td>8.416</td>
<td>8.416</td>
</tr>
<tr>
<td>R3</td>
<td>1.410</td>
<td>1.410</td>
<td>1.410</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R5</td>
<td>7.410</td>
<td>7.410</td>
<td>7.410</td>
<td>7.410</td>
<td>7.410</td>
</tr>
</tbody>
</table>

Table 10: Cost details of the optimum designs for the ex.2.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Vol.Conc. (m³)</td>
<td>2.84</td>
<td>2.87</td>
<td>2.27</td>
<td>2.89</td>
<td>2.85</td>
</tr>
<tr>
<td>Weight tot. (kg)</td>
<td>154.62</td>
<td>163</td>
<td>202.48</td>
<td>160.72</td>
<td>150.27</td>
</tr>
<tr>
<td>Cost Conc. ($)</td>
<td>113.50</td>
<td>114.84</td>
<td>90.90</td>
<td>115.75</td>
<td>113.93</td>
</tr>
<tr>
<td>Cost Ex. ($)</td>
<td>61.85</td>
<td>65.32</td>
<td>80.99</td>
<td>64.29</td>
<td>60.10</td>
</tr>
<tr>
<td>Cost Total ($)</td>
<td>175.35</td>
<td>180.16</td>
<td>171.89</td>
<td>180.04</td>
<td>174.03</td>
</tr>
</tbody>
</table>

Table 11: Optimum values of design variables for the ex. 2.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>X1</td>
<td>2.85</td>
<td>3.25</td>
<td>3.05</td>
<td>3.26</td>
<td>2.91</td>
</tr>
<tr>
<td>X2</td>
<td>1.40</td>
<td>1.44</td>
<td>1.56</td>
<td>1.47</td>
<td>1.46</td>
</tr>
<tr>
<td>X3</td>
<td>0.45</td>
<td>0.45</td>
<td>0.21</td>
<td>0.46</td>
<td>0.46</td>
</tr>
<tr>
<td>X4</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>X5</td>
<td>0.35</td>
<td>0.34</td>
<td>0.36</td>
<td>0.35</td>
<td>0.34</td>
</tr>
<tr>
<td>X6</td>
<td>2.65</td>
<td>2.83</td>
<td>2.71</td>
<td>2.88</td>
<td>2.61</td>
</tr>
<tr>
<td>X7</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>X8</td>
<td>0.75</td>
<td>0.31</td>
<td>0.60</td>
<td>0.20</td>
<td>0.64</td>
</tr>
<tr>
<td>R1</td>
<td>5.616</td>
<td>5.616</td>
<td>9.616</td>
<td>2.926</td>
<td>5.616</td>
</tr>
<tr>
<td>R3</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>R5</td>
<td>7.410</td>
<td>7.410</td>
<td>5.412</td>
<td>7.410</td>
<td>7.410</td>
</tr>
</tbody>
</table>

6.2 Case 2: Efficiency of steel and concrete classes on optimum CO₂ emission and cost

In this case, a new retaining wall having 4.0m height are optimized using the BBO and the SSO algorithms by considering...
minimizing the cost, minimizing the CO₂ emission and
minimizing both the cost and the CO₂ emission objective
functions. In each optimization test, the example problem is
optimized using different concrete and steel material types
which are described in Table 1. The unit weight of concrete and
steel are taken as 2300 kgf/m³ and 7850 kgf/m³ respectively.
Other parameters are same as the first example of the case 1.
Optimum cost, CO₂ emission and multi-objective function
values of all optimum solutions of the presented algorithms are
illustrated in Tables 13 and 14. According to the tables that
optimum costs which are obtained by considering different
objectives vary from 0.29% (for C27, S400 materials) to 3% (for C30, S500 materials) for the BBO algorithm; from 1.52%
(for C24, S400 materials) to 3.93% (for C30, S400 materials) for
the SSO algorithm. These differences are not significant.
However, the differences between maximum and minimum
values of optimum CO₂ emission are 3.44% (for C27, S400
materials) and 8.23% (for C30, S400 materials) for the BBO
algorithm; 3.71% (for C27, S400 materials) and 24.4% (for C30,
S400 materials) for the SSO algorithm. These are considerably
high. In addition there is not supremacy of any optimization
can be concluded in the tables.

<table>
<thead>
<tr>
<th>Objective</th>
<th>C24</th>
<th>C27</th>
<th>C30</th>
<th>C24</th>
<th>C27</th>
<th>C30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>241.7</td>
<td>264.1</td>
<td>269.4</td>
<td>241.2</td>
<td>277.1</td>
<td>281.8</td>
</tr>
<tr>
<td>CO₂</td>
<td>804.4</td>
<td>815.3</td>
<td>888.1</td>
<td>781.7</td>
<td>792.2</td>
<td>866.0</td>
</tr>
<tr>
<td>Multi</td>
<td>1040.1</td>
<td>1079.4</td>
<td>1157.5</td>
<td>1022.9</td>
<td>1069.3</td>
<td>1147.8</td>
</tr>
<tr>
<td>Cost</td>
<td>244.6</td>
<td>271.7</td>
<td>274.0</td>
<td>241.9</td>
<td>279.5</td>
<td>282.6</td>
</tr>
<tr>
<td>CO₂</td>
<td>743.0</td>
<td>752.1</td>
<td>853.3</td>
<td>754.7</td>
<td>764.1</td>
<td>807.3</td>
</tr>
<tr>
<td>Multi</td>
<td>987.6</td>
<td>1023.8</td>
<td>1127.3</td>
<td>996.6</td>
<td>1043.6</td>
<td>1089.9</td>
</tr>
<tr>
<td>Cost</td>
<td>241.8</td>
<td>269.0</td>
<td>269.7</td>
<td>241.8</td>
<td>277.2</td>
<td>281.9</td>
</tr>
<tr>
<td>CO₂</td>
<td>744.2</td>
<td>753.6</td>
<td>857.3</td>
<td>760.1</td>
<td>765.9</td>
<td>810.1</td>
</tr>
<tr>
<td>Multi</td>
<td>986.0</td>
<td>1022.6</td>
<td>1127.0</td>
<td>1001.9</td>
<td>1043.1</td>
<td>1092.0</td>
</tr>
</tbody>
</table>

Table 12: The objective function values of the BBO algorithm
for case 2.

Distribution of optimum values with respect to different
materials in case of the minimizing cost and the minimizing CO₂
objectives are plotted in Figures 7, 8 respectively. It is
concluded from the figures that the lowest cost value is
obtained using the C24 and S400 materials. The second lowest
cost is obtained using the C24 and S500 materials which is close
to the lowest cost value of the C24 and S400 materials.
Moreover, the lowest CO₂ emission is obtained using the C24
and S400 materials. In addition, the optimum cost and CO₂
values increase when concrete strength is increased. On
the contrary, any relationship between the steel class and
the optimum cost values cannot be defined.

Concrete and steel costs/CO₂ emissions of the optimum designs
are illustrated in Figures 9-14 respectively. Five inferences are
concluded from these figures. The first one is: concrete cost rate
is generally higher than steel cost rate. The second one is: steel
cost rate generally increases in higher steel classes. The third
one is: steel cost rate for minimize cost objective functions is
higher than steel cost rate for minimize CO₂ emission objective
function. The fourth one is: the concrete CO₂ emission rate
for minimize cost and multi objective functions is lower than
the concrete CO₂ emission rate for minimize cost objective
function. The fifth one is: the CO₂ emission of concrete much
higher than CO₂ emission of steel.
7 Conclusion

In this study, the BBO and the SSO algorithms are proposed and utilized to calculate the cost and the CO2 emission for the RC retaining wall design through investigation of two main subjects. First is the comparison performance of the meta-heuristic search algorithms for retaining wall design problems and the second is the optimization of the retaining wall under environmental considerations.

In first case, the 3.5 m and 5.2 m height retaining walls which are previously used in the literature, are optimized by considering minimizing the cost objective functions and taking unique material properties. The obtained results are compared to literature studies. In the first example, the BBO algorithm shows the best performance and the SSO algorithm has third best performance. Whereas, the SSO algorithm shows the best performance and the BBO algorithm has fifth best performance. Therefore, the supremacy between the SSO and BBO algorithms cannot be defined. However, it can be concluded that, the presented algorithms are powerful and efficient in finding the optimum solution for optimum cost design of RW problems.

In the second case, the new example (4.0 m height retaining wall) is optimized using the presented algorithms by considering minimizing the cost, minimizing the CO2 and minimizing the weighted aggregate of the cost and the CO2 objective functions. Three concrete material types (C24, C27 and C30) and two steel material types (S400 and S500) are used. In total, the design example is optimized thirty six times by taking different objectives and different materials. According to obtained results from these runs, six main outcomes are obtained. The first outcome is that the optimization of the retaining wall considering the minimizing CO2 emission objective function does not have a material influence on the optimum cost of the retaining wall. Therefore, the minimizing CO2 emission objective function can be used in the cost optimization problem. The second outcome is that when lower material classes (especially concrete class) are used, better optimum cost values and optimum CO2 emissions are obtained. In summary, if lower class materials are used, lower cost and CO2 emissions are obtained. The third outcome is: usage of higher strength concrete increases steel material usage. The fourth outcome: concrete material cost constitute majority of total cost and amount of concrete has huge percentage in total CO2 emission. The fifth outcome: steel cost rate increases when the minimizing CO2 emission and multi-objective functions are used. The sixth outcome: CO2 emission rate of concrete is decreases when the minimizing CO2 emission and multi-objective functions are used.

8 List of symbols

$A$: Cross sectional area of the structural material,
$ACoeff$: Absorption coefficient,
$A_{min}$: Minimum reinforcement area,
$A_{max}$: Maximum reinforcement area,
$B0$: Attractiveness at original location,
$C$: Summation of constraint violations,
$C_s$: Unit cost of steel,
$d_b$: Diameter of the hooked bar,
C\(_c\): Unit cost of concrete,

E\(_{\text{CO}_2}\): \(\text{CO}_2\) emission of the structural material,

f: Value of the objective function,

f\(_c\): Female spider,

f\(_{\text{high}}\): Objective function of worst spider,

f\(_{\text{low}}\): Objective function of best spider,

f\(_p\): Penalized value of the objective function,

g\(_i\): \(i\) \text{th} constraint function,

H: Height of stem,

HMS: Harmony memory size,

HMCR: Harmony memory considering rate,

I: Maximum immigration rate,

k: Iteration number,

L\(_{\text{dev}}\): Minimum development length,

L\(_{\text{hook}}\): Minimum hook development length,

M\(_d\): Design moment,

m\(_j\): Male spider,

m\(_{\text{max}}\): Maximum mutation probability,

M\(_u\): Ultimate resistance moment,

N: Size of population,

NC: Number of constraint functions,

N\(_{\text{dom}}\): Number of dominant male spiders,

NDV: Number of design variables,

N\(_f\): Number of female spiders,

NFF: Number of firefly,

N\(_m\): Number of male spiders,

N\(_{\text{material}}\): Number of materials,

N\(_s\): Number of spiders,

PAR: Pitch adjusting rate,

PF: Female movement parameter between \(0, 1\),

P\(_{\text{max}}\): Maximum number of species,

r: Random number between \(0, 1\),

round: Function which rounds to the value of the nearest integer,

RP: Randomness parameter,

RP\(_{\text{min}}\): Minimum randomness parameter,

RP\(_{\text{max}}\): Maximum randomness parameter,

R1: First vertical steel reinforcement in the stem,

R2: Second vertical steel reinforcement in the stem,

R3: Horizontal steel reinforcement in the toe,

R4: Horizontal steel reinforcement in the heel,

R5: Vertical steel reinforcement of key,

S\(_b\): Best spider,

S\(_{\text{max}}\): Maximum number of species,

S\(_{\text{med}}\): Clear spacing,

S\(_{\text{max}}\): Maximum clear spacing,

S\(_o\): Worst spider,

S\(_e\): Equilibrium number of species,

V\(_c\): Volume of concrete per unit length of the Wall,

V\(_d\): Design shear force,

vib\(_{ij}\): Vibration between the \(i\) \text{th} spider and the nearest spider,

vib\(_{ib}\): Vibration between the \(i\) \text{th} spider and the best spider,

vib\(_{if}\): Vibration between the \(i\) \text{th} spider and the nearest female spider,

V\(_u\): Ultimate shear capacity,

w\(_b\): Weight of the best spider,

w\(_c\): Weight of the nearest spider,

w\(_f\): Weight of the nearest female spider,

w\(_{\text{med}}\): Weight of the median spider,

W\(_{\text{st}}\): Weight of steel per unit length of the Wall,

X: Vector of design variables,

x\(_b\): Design variable of the best spider,

x\(_c\): Design variable of the nearest spider,

x\(_f\): Design variable of female spider,

x\(_l\): Lower boundary of design variable,

x\(_m\): Design variable of male spider,

x\(_u\): Upper boundary of design variable,

X1: Width of the base,

X2: Toe projection,

X3: Thickness at the bottom of the stem,

X4: Thickness at the top of the stem,

X5: Thickness of base slab,

X6: Distance from toe to the front of the base shear key,

X7: Width of the key,

X8: Depth of the key,

\(\alpha\): Random number between \(0, 1\).
\( \beta: \) Random number between \( \{0, 1\} \),
\( \&: \) Random number between \( \{0, 1\} \),
\( \varepsilon: \) Penalty coefficient,
\( \zeta_{\text{cost}}: \) Non-negative weight coefficient of cost,
\( \zeta_{\text{CO}_2}: \) Non-negative weight coefficient of \( \text{CO}_2 \),
\( \lambda: \) Immigration rate,
\( \mu: \) Emigration rate,
\( \rho: \) density of the structural material.

9 References


