

## ABSORPTION OF ENERGY BY FRUSTA

**A. A. ALGHAMDI\***

**A. K. EL-KALAY\***

**T. M. ABU-MANSOUR\***

**M. AKYURT\***

*SUMMARY: Deformable energy absorbers are briefly introduced, and their relevance in crash protection systems is stressed. A general review of literature is presented on energy absorption by tubes of various cross sections. Work on quasi-static and impact-loaded hollow and capped, spun aluminum frusta is described. Experimental findings are reported that suggest that certain frusta may be used as active elements, and that in this respect frusta may be preferable to "constant-force energy absorbers" such as tubes. It is established for the inversion of stacked multi-element absorbers that the energy absorbed per gram is not affected by the series arrangement. It is further concluded that the amount of energy absorbed in inward inversion is more than twice that of outward inversion. It is pointed out that while tube inversion requires dies, the length of tube to be inverted has limitations, and that normally tube inversion is limited to about half the original tube, height, none of these shortcomings were encountered during the inversion of aluminum frusta.*

*Key Words: Energy frusta, energy absorbers.*

### INTRODUCTION

Energy absorbers are systems that convert kinetic energy into other forms of energy, such as pressure energy in compressible fluids, elastic strain energy in solids, and plastic deformation energy in deformable solids. The converted energy may be reversible, as in pressure energy in compressible fluids, and elastic strain energy in solids, or irreversible, as in plastic deformation. The process of conversion for plastic deformation depends, among other factors, on the magnitude and method of application of loads, transmission rates, deformation displacement patterns, and material properties (1).

Familiar deformable energy absorbers include steel drums, liquid and sand filled cells, including sand bags, barrels made of plastics, vermiculite concrete with voids, honeycomb cells filled with polyurethane foam, and tubular elements in a variety of shapes and arrangements. The active absorbing element of an energy absorption system can assume several common shapes such as tubes, honeycombs, frusta, strips, and rods.

The predominate domain of applications of deformable energy absorbers is that of crash protection. Such systems are installed in high-risk environments with potential injury to humans or damage to property. The

aim is to minimize by controlling the deceleration pulse during impact. This is achieved by extending the period of dissipation of the kinetic energy of the system over a finite period of time. Cushioning devices on vehicle bumpers, crash retarders in emergency systems of lifts, and some crash barriers used as road blocks are everyday examples. Energy absorbers are likewise utilized during the installation of machinery, including household appliances, to minimize foundation-related vibrations.

Development of mechanical devices that help dissipate kinetic energy at predetermined rates has been receiving substantial attention for several decades (2). When designing a collapsible energy absorbing (dissipating) device, the prime aim is to absorb the majority of the kinetic energy of impact within the device itself in an irreversible manner, and in such a way as to ensure that injuries to humans and damages to equipment are minimum. The absorbing system must exhibit force-deflection characteristics which result in decelerations that are within allowable limits. These limits may be dictated either by human tolerance levels or by the maximum force the structures themselves can withstand (3).

Axisymmetrical and circular shapes provide perhaps the widest range of all choices for use as absorbing elements because of their favorable plastic behavior under axial crumbling forces, as well as their common occurrence as structural elements.

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\* From Department of Mechanical Engineering, King Abdulaziz University, P.O. Box 9027, Jeddah 21413, Saudi Arabia.

### Modes of deformation

The study of deformation of tubular energy absorbers in general falls into two main categories; lateral, and axial loading. Investigations often lead to accounting for geometrical changes, interactions between modes of collapse, as well as strain hardening, and strain rate effects.

Johnson and Reid (4) identified the dominant modes of deformation in simple structural elements in the form of circular and hexagonal cross-section tubes, and piping beams, when these elements were subjected to various forms of quasi-static loading. They described the load-deformation characteristics of a number of these elements. Jones (3) reviewed work done on the dynamic plastic behavior of structures.

Simple rigid-plastic analysis for the lateral loading (or flattening) of tubular beams was given by Deruntz and Hodge (5), which indicated that the force deflection relationship is a function of the length, diameter, and thickness of the tube. The flattening between flat plates was examined by Reid *et al.* (1), and two-points loading was examined by Reid and Ball [as reported by Reid (6)] Reddy *et al.* (7), and Carney and Pothen (8) reported on energy absorbed in braced cylindrical shells. Reid *et al.* (9) described the modes of deformation and the behavior of tubular rings under static and dynamic loads. Watson *et al.* (10) studied local loading of tubes, and reported a method of energy calculation which provided reasonable agreement with the experimental data.

Thin-walled absorbers having symmetrical cross sections may collapse in the concertina or the diamond mode when subjected to axial loads. The collapsing of such components by splitting or by inversion is also conceivable. Thus Ezra and Fay (11) identified the combined modes of axial splitting and subsequent curling of the split ends of the tubes as an efficient means of energy dissipation in absorber elements. Stronge *et al.* (12) examined the splitting of square-section tubes, where two modes of splitting were identified, with and without inversion.

Wilsea *et al.* (14) studied the indentation behavior of polyurethane foam under axial load. Abramowicz and Wierzbicki (15) discussed the crushing of foam-filled columns, and suggested a theoretical model. Sand-filled tubular beams under a central load were found to be effective for use as energy absorbers (6).

It was noted that strain rate sensitivity of the absorber material must be taken into account by determining the dynamic yield and ultimate stresses of the material (16). In this connection it was reported that the effect of strain hardening appears to be small unless the material has very strong hardening properties (3).

### Axial loading of tubular components

The behavior of thin tubes (diameter  $D$  / thickness  $t > 50$ ), with circular and square cross-section, when subject-ed to axial loads, has been of particular interest since the pioneering work of Pugsley in 1960 (17). In fact circular tubes under axial compression are reported to be the most prevalent components in energy absorber systems (18). This is because the circular tube provides a reasonably constant operating force. Furthermore, circular tubes have comparatively high energy absorbing capacities, and stroke length per unit mass. For example, in comparing lateral with axial compression, the axial buckling mode has a specific energy absorbing capacity which is approximately ten times that of the same tube when compressed laterally between flat plates (18). Moreover, practically all wall material in a tube can be made to participate in the absorption of energy by plastic work in axial loading.

Experimental observations show that thick-walled cylinders buckle in concertina (axisymmetric) mode of deformation, whereas thin-walled cylinders buckle in diamond (non-axisymmetric) mode of deformation. For large values of  $D/t$ , where a diamond-fold mode of deformation tends to occur, the number of lobes increases with  $D/t$  ratio. Pugsley (19) and Batterman [as reported by Tvergaard (20)] observed the transition point from concertina mode to diamond mode to be at  $D/t=90$  while Lee, according to Tvergaard (20), suggested this point to be at  $D/t=30$ . Tvergaard (20) proposed a theoretical model for the prediction of the transition point. His result indicated a transition point at a value of  $D/t$  somewhere in the range 50-100.

Alexander (21) presented a rigid-plastic analysis for concertina mode of deformation. Pugsley and Macaulay (17), and Pugsley (19) proposed a semi-empirical rigid-plastic theoretical estimate of the mean axial load for diamond mode of deformation. Wierzbicki gave an approximate expression for diamond mode of deformation (16).

Jones and Abramowicz (16) proposed relationships for finding the average crushing load for both the diamond and the concertina modes of deformation. Wierzbicki and Abramowicz (22) analyzed the crushing of thin-walled structures made from plate elements. Johnson *et al.* (23) considered an inextensional mode of deformation, and calculated the corresponding mean crushing load.

One of the interesting phenomena that occurs in energy absorber elements is that of tube inversion, i.e., the turning inside-out or outside, in of a thin tube made of a ductile material. A significant feature of this mode of deformation is the constant inversion force that can be

obtained for a uniform tube (13), giving rise to a square retardation pulse. Tube inversion cannot be affected unaided, however, since a minimum of one die is required for the process. Another important drawback of tube inversion is that there is a maximum tube height, as determined by the mean diameter and the wall thickness, which if exceeded, failure occurs due to buckling and not due to inversion. Furthermore, inversion is limited to about half the original tube height unless special, costly arrangements are incorporated.

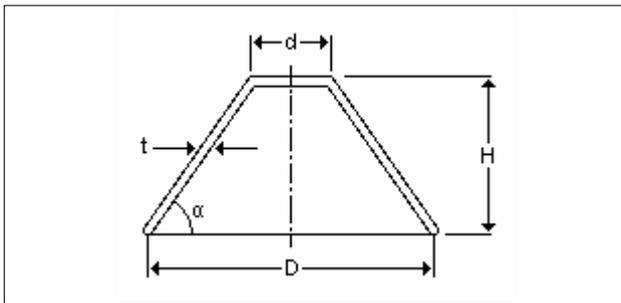


Figure 1: A capped frustum.

**Thin-walled Frusta**

Frusta are truncated circular cones, as illustrated in Figure 1. Frusta are employed over a wide range of applications, especially in the domains of aerospace and armaments. Common examples occur in the nose cones of missiles and aircraft.

Literature on the utilization of frusta for dissipation of energy is meager. The Frustum was first studied in this context in 1970 by Postlethwaite and Mills (24). They used Alexander's extensible-collapse analysis for the concertina mode of deformation (20) for hollow cones and frusta made of rigid, perfectly plastic material.

Mamalis *et al.* (25) discussed the crumbling of aluminum frusta under quasi-static compression. They proposed empirical relationships for both the concertina and the diamond modes of deformation. Mamalis *et al.* (26) extended their study to include mild steel. They proposed that the deformations in frusta be classified as a) concertina, b) concertina-diamond, and c) diamond. Mamalis and associates further proposed theoretical models for predicting the mean crushing load (27), and the plastically dissipated energy (28) for axially loaded cones and frusta.

In what follows, results of experimental work conducted on the free (inward) inversion and outward inversion (crushing) of capped, spun aluminum frusta are presented.

**EXPERIMENTAL**

Preliminary trials of (inward) inversion were conducted on aluminum and mild steel frusta that were produced by the processes of a) machining from solid stock, b) forming and welding, and c) spinning. Frusta produced by spinning were found to yield the most consistent results. Consequently all frusta for the experimental program were manually spun from blanks of commercial aluminum sheets 0.5, 1.0, 1.5, and 2.0 mm in thickness. Three different mandrels (Figure 2),

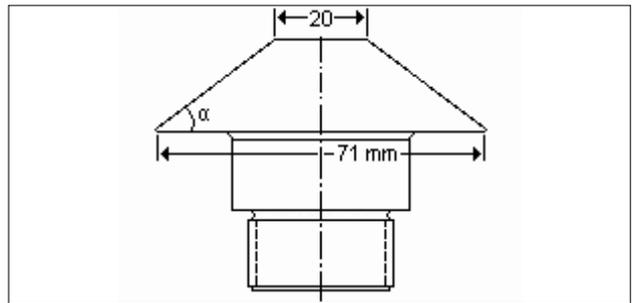


Figure 2: Mandrel for spinning.

featuring angles  $\alpha$  of 30, 45, and 60°, were manufactured for the spinning process. As they were not subjected to subsequent heat treatment, the frusta produced by cold spinning featured locked-in residual stresses. Moreover, axial variations were noticeable in the thickness of the shell of the frusta.

A total of 58 frusta, featuring different thicknesses and apex angles were subjected to various operations of inversion. The program involved the use of twelve different sizes of aluminum frusta (Table 1) for inversion and crumbling (outward inversion). Additional tests were conducted to investigate the effect on inversion of a) quasi-static and impact loading, b) multiple inversions, and c) stacking.

Tests were conducted by the use of a 50-ton Instron universal testing machine (UTM) as well as a falling

Table 1: Frusta sizes.

	Apex Angle $\phi$ - deg		
	30	45	60
Shell Thickness (mm)	0.5	0.5	0.5
	1.0	1.0	1.0
	1.5	1.5	1.5
	2.0	2.0	2.0

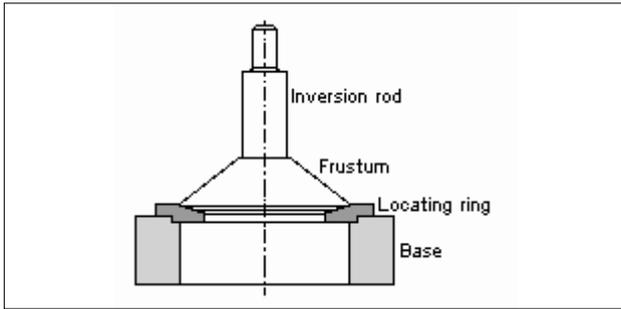


Figure 3: The jig for (inward) inversion.

weight hammer (FWH) of 10 m/s striking speed. Two special jigs, one for (inward) inversion, and the other for outward inversion (crumbling), were manufactured and utilized in conjunction with the UTM and the FWH. The inversion jig (Figure 3) consisted of an inversion rod, specimen holder, and a base. The rod was clamped by the upper jaw of the UTM, and the base rested on the lower jaw. The stepped specimen holder allowed the testing of frusta of several sizes and types. The setup for outward inversion comprised essentially two parallel plates. The upper plate was gripped by the upper jaw of the UTM, and the lower plate rested on the lower jaw. Both jigs were utilized also with the FWH, in which case the inversion rod and the upper plate of the inversion jig were simply placed on the specimen.



Figure 5: Frusta after inversion.

RESULTS AND DISCUSSION

Figure 4 is a typical load-displacement curve for spun frusta ( $\phi=60^\circ$ ) in the inversion mode. The load rises quasi-linearly from the start up to about point (a). The force at point (a) represents the load of instability. Up to this point the deformation is recoverable, i.e., elastic, and beyond which plastic behavior sets in.

The zone between (a) and (b) is a zone of incubation, within which the cap of the frustum is deformed in such a manner as to facilitate the inversion type of deformation. Point (b) signals completion of the devel-

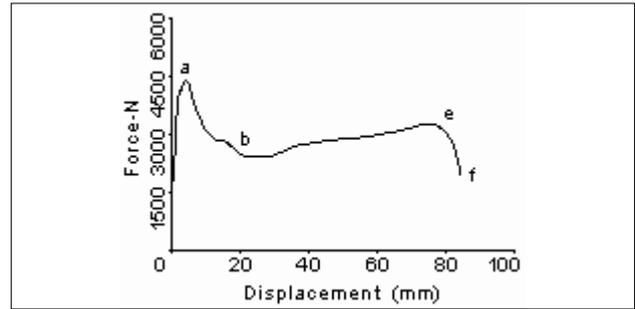


Figure 4: Variation of applied force during inversion.

opment of the inversion zone. Inversion then proceeds towards the big end of the frustum, until point (e) is reached. The increase in the inversion force from (b) to (e) may be attributed to the progressive increase in the volume of the deformation zone with the increasing  $D/t$  ratio.

Point e in Figure 4 signals the termination of the inversion zone, the bending front having reached the vicinity of the free large end of the frustum. From point e to f the free end of the frustum is flattened parallel to the shoulder of the jig base. Figure 5 exhibits samples of frusta that were inverted in the manner described.

In order to assess the effect of speed on the process of inversion, identical frusta were tested at cross-head speeds of 2, 20 and 200 mm/min. Additional tests were conducted on the FWH facility for both inward and outward inversion. As all specimens in these tests behaved as in quasi-static tests, it was concluded that the process of inversion is not affected by strain rate.

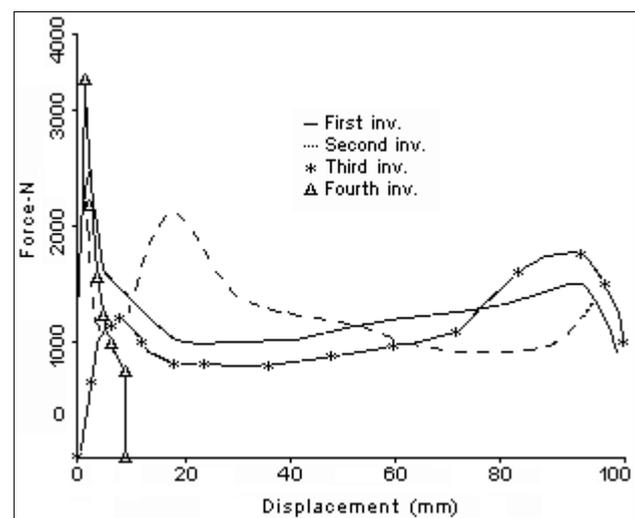


Figure 6: Variation of applied force during inversion.

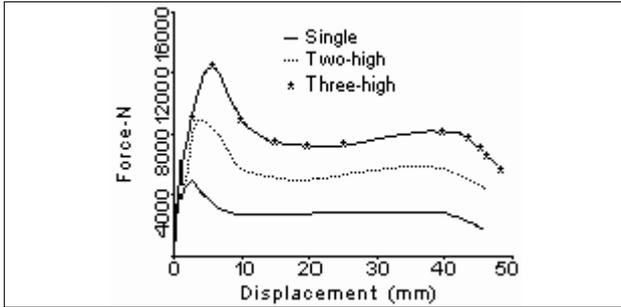


Figure 7: Variation of applied force during inversions of stacked frusta.

The possibility of re-using the inverted frusta was investigated next. Several tests were conducted for inversion and re-inversion of frusta. Typical results from one such experiment are summarized in Figure 6. It became clear that it is possible to invert, re-invert and then again invert an aluminum frustum. All specimens failed, however, during the fourth inversion. The particulars of the frustum of Figure 6 were  $D=73$  mm,  $d=22$  mm,  $H=46$  mm, and  $t=0.8$  mm, mass=16.1 grams, and  $\alpha=60^\circ$ .

Further inversion tests were conducted on identical frusta when stacked two, three, and four high. Representative load-displacement curves for a single frustum, as well as for stacks consisting of two and three frusta are shown in Figure 7. Data on average inversion force, total absorbed energy, absorbed energy per frustum, absorbed energy per gram, and the peak force are summarized in Table 2.

A number of tests were undertaken for the outward inversion (crushing) of frusta between two parallel plates. The load-displacement curve for a typical test is displayed in Figure 8 for a specimen of  $D=73$ ,  $d=22$ ,  $t=0.9$ ,  $H=26.5$  mm,  $\alpha=45^\circ$ , and mass=13.4 grams. As in the case of tube inversion (13), elastic behavior is observed (Figure 8) up to point (a). Point (b) is the point of plastic instability, at which the rim of the lower end deforms into the shape of a funnel. The decrease in the inversion force between points (b) and (c) signifies the development of the free-radiusing of the lip at this end.

Table 2: Inversion of stacks of frusta.

Arrangement	Displacement (mm)	Max force (N)	Energy (J)	Energy/frustum (J)	Specific Energy (J/gm)
Single	46.0	4360	124	124	6.45
Double	46.5	8160	248	124	6.28
Triple	48.5	11940	370	123	6.18

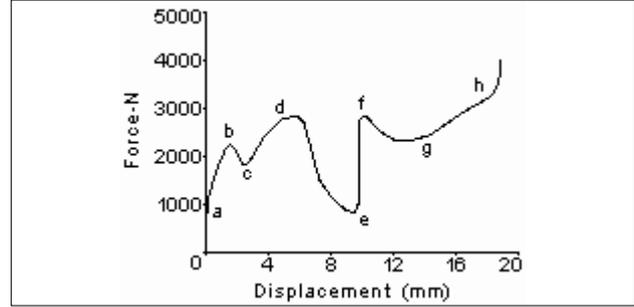
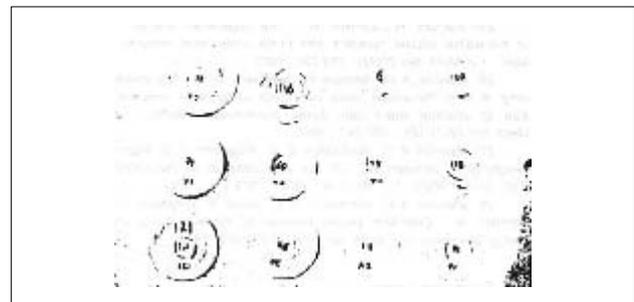


Figure 8: Variation of applied force during outward inversion.

The force increases again after (c), as material is being pushed radially outward while the diameter of the deformation zone decreases. At point (d) the zone of outward inversion is fully developed, and the lip of the inverted rim curls upward from the lower plate. The section of the curve from (d) to (e) depicts a rapid decrease in the inversion force, resulting from the progressively diminishing volume of deformation inherent to the shape of the cone.

The sharp rise represented by the (e)-(f) sector develops when the rising lip of the inverted rim meets the upper plate, which forces it to bend (re-invert) downwards, in the direction opposite to the direction of the

Figure 9: Frusta after outward inversion.



first inversion. This is followed by the development of a second inversion zone (f to g), where residual stresses from the first inversion are thought to be the cause of the lowering of the load in this sector. Both inversion zones are operative in the region from (g) to (h), while the frustum is being crushed flat at point (h). Figure 9 shows frusta after undergoing outward inversion.

CONCLUSIONS

To compare the inversion characteristics of frusta with those of thin tubes, it must be pointed out that tube inversion cannot be executed unaided, since a mini-

imum of one die is required for the process. A second and important drawback of tube inversion is that there is a maximum tube height that is determined by the mean diameter and the wall thickness, which if exceeded, failure occurs due to buckling and not due to inversion. Furthermore, inversion is limited to about half the original tube height unless special, costly arrangements are incorporated. None of these shortcomings are encountered during the inversion of frusta.

Spun aluminum frusta (capped) produced results that were repeatable during quasi-static as well as dynamic inversion. Many specimens were inverted, some with thickness less than 0.55 mm, with no specimen rupturing during the process. The mode of deformation was always consistent with the type of jig in use. In fact it was found that an absorber in the inversion mode may be inverted several times, indicating that it is possible to re-use the same absorber. Since all specimens in the impact tests behaved as in the quasi-static tests, it is concluded that within the experimental range of the impact speeds, the process of inversion is not affected by the speed of inversion.

In frusta with  $\phi=60^\circ$ , the inversion force increased gradually with displacement. This indicates that such frusta may be classified as "increasing-force energy absorbers", and hence may be recommended as active elements. In this respect they may be preferable to "constant force energy absorbers" such as tubes, due to the ensuing deceleration characteristics.

The results of experiments on the inversion of stacked frusta establish that the inversion of multi-element absorbers that are made of a number of frusta in series is possible. The results also indicate that for the inversion of stacks of frusta, the energy absorbed per gram is not affected by the series arrangement. Furthermore it is demonstrated that the absorber capacity increase linearly with the number of frusta in series.

It is concluded from a comparison of the results of tests involving inward inversion with dose of outward inversion that the overall amount of energy absorbed in inward inversion is more than twice that of outward inversion. This is because the "stroke" in inward inversion is considerably longer than in outward inversion, even though the average force in the latter is higher.

#### ACKNOWLEDGEMENT

The authors would like to thank Professors S. T. S. Al-Hassani and A. R. Manna for useful discussions and S. S. Habib for the use of testing and computer facilities.

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Correspondence:

A. A. Alghamdi  
 Department of Mechanical Eng.  
 King Abdulaziz University,  
 P.O. Box 9027,  
 Jeddah 21413,  
 SAUDI ARABIA.