COMPUTER-AIDED ANALYSIS OF MECHANISMS BY INVERSION

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SUMMARY: Inversion is presented as a method of analysis in mechanisms. Analysis of displacements is taken up first, and the relevant relationships are stated. This is followed by the analysis of kinematics. Use is made of the AL-YASEER software package for the analysis of mechanisms over a complete cycle of events. The slider-crank mechanism and its inversions, as well as the fourbar linkage are employed for demonstrating the utility of the relationships.

Inversion is applied next to mechanisms of both low and high complexity, and examples are provided for the analysis of position as well as kinematic analysis over complete cycles. Bolstered with the method of kinematic coefficients, it is concluded that inversion represents a potentially useful method for the analysis of complex mechanisms.

Key Words: Analysis, Complex, Computer-Aided, Inversion, Mechanism.

INTRODUCTION

In earlier publications Mannaa and associated (1-6) introduced the software package "AL-YASEER", and illustrated its use. The concept of a virtual, flexible link was likewise expounded (7,8). It was confirmed that AL-YASEER, compounded with a flexible virtual crank, can indeed be used successfully to tackle many cases of complicated mechanisms and machinery. What is expected from the user during the solution process was demonstrated to involve little more than the supplying of input, and certain equivalency statements. The computer then caries out all of the associated tedious work and delivers a comprehensive analysis of the problem at hand.

In what follows the focus is on the study of a still different class of problems, the so called "kinematically complex mechanisms". These are mechanisms that accommodate one or more multipaired floating links, each of which is connected to at least three other moving links (9). Figures 1 to 4 are examples of complex linkages. The floating member BCD in Figures 1, 2 and 4 is completely constrained by two of the three attached links, and only one radius of path curvature is unknown. Thus the four-

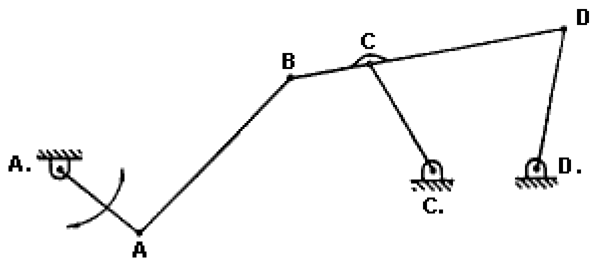

Figure 1: Complex mechanism CM-1.

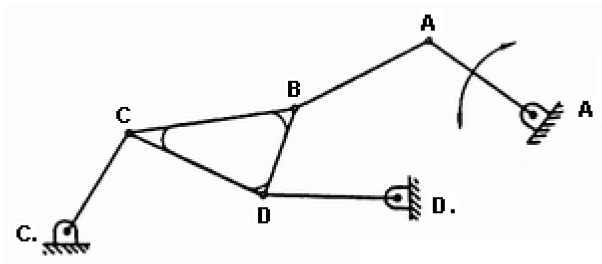

Figure 2: Complex mechanism CM-2.

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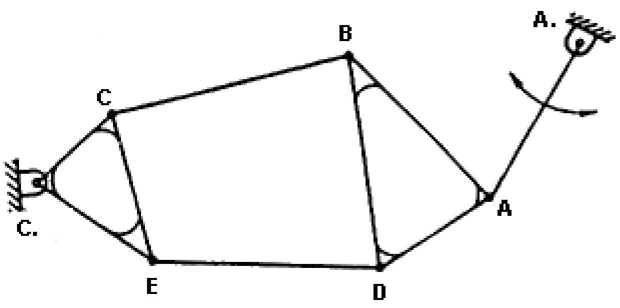


Figure 3: Complex mechanism CM-3.

bar linkages CoCDDo in Figures 1 and 2 and CoCDAo in Figure 4 completely constrain the motion of BCD. Consequently these mechanism (CM-1, CM-2 and CM-4) are said to have a low degree of complexity. These as well as the rest of the drawings in this communication are not drawn to scale.

If mechanism CM-1 is driven by member AoA, it becomes complex. When driven by link CoC or DoD, however, the same mechanism becomes simple, and is amenable to AL-YASEER. Similar observations can be made on mechanisms CM-2 and CM-4.

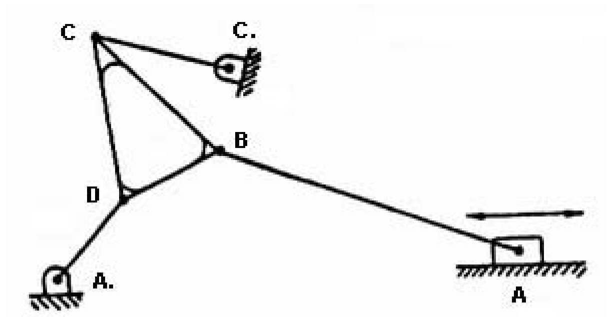


Figure 4: Complex mechanism CM-4.

Considering mechanism CM-3, shown in Figure 3, it is noted that all three of the attached links are essential for link ABD to be constrained. This mechanism, therefore, is said to have a high degree of complexity; it is complex no matter which of the links that are attached to the frame is used to drive the mechanism.

Several techniques have been developed (10-13) for the kinematic analysis of complex linkages. These require graphical constructions. Resorting to the use of loop-closure equations, as discussed by Akyurt (8), will yield suitable results; the degree of diligence expected from the user, and the amount of labor involved, however, are generally too excessive to warrant this choice.

It would be desirable to develop algorithms and other computer techniques that allow automatic computation with minimum contribution from the user. As a part of the efforts to this end, the case of mechanisms of low complexity was taken up in prior communications (14,15). The discussion below focuses, therefore, on exploring the potentiality of the method of inversion for facilitating the analysis of linkages with high degree of complexity.

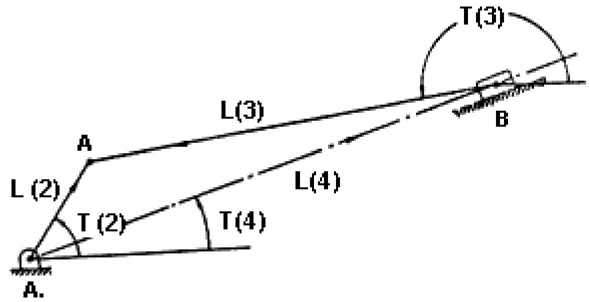


Figure 5: The centric slider-crank mechanism.

INVERSION

Inversion is the process where the fixed member in a kinematic chain is set free, and another link is kept fixed instead. It is well known that relative displacements, velocities and accelerations in a given chain or mechanism are not altered when the mechanism is inverted. This observation leads to the interesting possibility analyzing a mechanism in a certain inversion, and then proceeding to a different inversion of the same mechanism, and the ensuing deduction of the corresponding kinematic data for the new case. This would be of special interest in cases where a kinematic analysis may be simple in a certain inversion, and not-so-simple for the inversion under investigation.

The centric slider-crank mechanism is considered below for the purpose of expounding the basic concepts. The nomenclature used is consistent with those in Prog 1 and Prog 3 of AL-YASEER.

KINEMATIC ANALYSIS OF A FISC MECHANISM BY INVERSION

Consider the centric slider-crank mechanism of Figure 5, where the inclinations of the crank AoA and the cylinder axis are T(2) and T(4), respectively. One relative angle between the crank and the cylinder axis is

relative angle BaoA= T(2) - T(4) [a]

For a slider-crank mechanism with fixed cylinder axis, we may differentiate Eqn [a] to obtain relative velocities and accelerations:

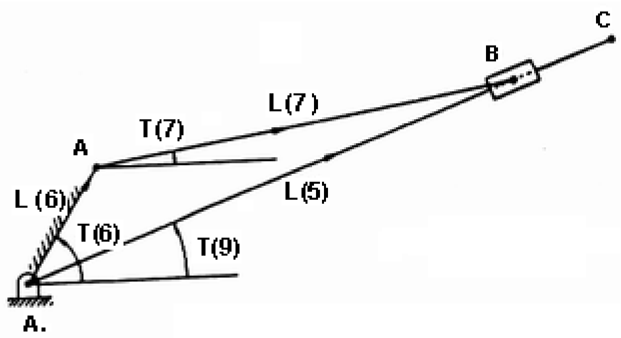


Figure 6: The first inversion of the slider-crank (FISC).

relative velocity = W(2) [b]
relative acceleration = A(2) [c]

Proceding in a similar manner, a relative angle at B between the connecting rod AB and the cylinder axis may be expressed:

relative angle AoBA = T(3)- T(4) [d]
relative velocity = W (3) [e]
relative acceleration = A (3) [f]

One of the inversions of the slider-crank mechanism is shown in Figure 6 (FISC), where member AoA is fixed, and the cylinder axis is released. The resulting mechanism is shown in its usual AL-YASEER markings for a FISC.

We now record, for the FISC mechanism, the relative angles and their derivatives at the same points and in the same manner as was done for the slider-crank mechanism.

At Ao: relative angle = T(6) - T(9) [g]
relative velocity = -W(4) [h]
relative acceleration = -A (4) [i]

At B: relative angle = [T(7)-180] - T(9) [j]
relative velocity = W(0) - W(4) [k]
relative acceleration = A(o) -A(4) [l]

If relative displacements and their time derivatives are to remain unchanged after inversion of a given mechanism, we must require that, at any given configuration,

[a] = [g]: T(2) - T(4) = T(6) - T(9) [1]
[b] = [h]: W(2) = -W(4)
[c] = [i]: A(2) = -A(4)
and
[d] = [j]: T(3) - T(4) = T(7) - 180 - T(9) [2]
[e] = [k]: W(3) = W(0) - W (4)
[f] = [l]: A(3) = A(0) - W(4)

Suppose now that, we have at our disposal, a position as well as kinematic analysis of the slider-crank mechanism of Figure 5. It is possible then, to deduce the position as well as kinematic data of the FISC of Figure 6, by the use of Eqns 1 and 2 as follows. Since T(2) and T(4) are

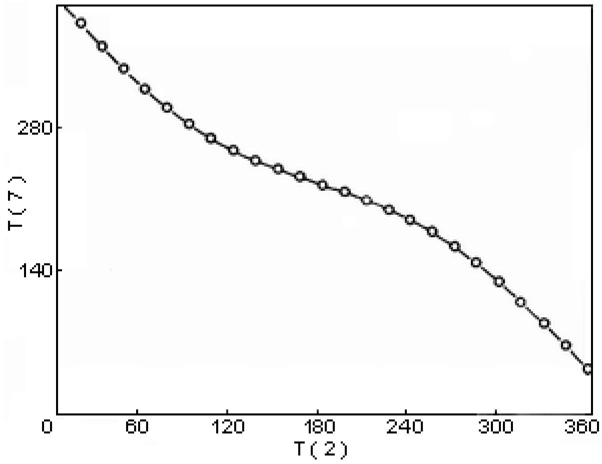


Figure 7: Variation of T(7) with crank angle T(2).

known for the slider-crank mechanism, and T(6) are known for the slider-crank mechanism, and T(6) is known for the FISC, it follows from Eqn 1 that

T(9) = T(6) + T(4) - T(2)

and upon differentiation,
W(4) = -W(2)
A(4) = -A(2)

Also from Eqn 2, for known values of T(3), T(4), and T(9)

T(7) = T(3) + T(9) - T(4) + 180
and W(0) = W (3) + W (4)
A(0) = A(3) + A(4)

Example 1: Given for a slider-crank mechanism (Figure 5) that T(4) = .1, L(2) = . 2, L(3) = . 4m, and uniform tawafwise (counter-clockwise) crank speed of 35 rad/s, a) Find T(3) and W(3) for a complete cycle, b) Determine the corresponding T(9) and W(4) for the FISCH (Figure 6) that is obtained by inversion of the same mechanism such that T(6) = 45.1^{0} C) Check the results for part (b) by computing T(9), and W(4) by calling Prog 3 of AL-YASEER.

Solution: The program as well as sample solutions are tabulated in Table 1. Columns 4 and 5 of the table show T(9) and W(4) as computed by Prog 3 (steps 5030 - 5040), and columns 6 and 7 show W(4) and T(9) as computed by the use of Eqns 1 and 2 and their derivatives

Figure 8: The four barlinkage with AoBo fixed.

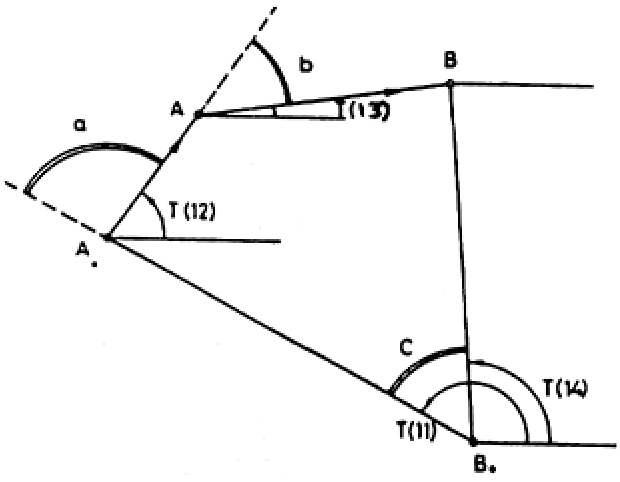


Table 1: Kinematic Analysis of Inverted Slider-Crank Mechanism.

```
5Ø1Ø  T (4)= . 1:   L (2)= . 2::   L (3) = . 4:   W (2) = 35 :   L (6) =
      L(2) :   T (6)= 45 . 1 :    L (7) = L (3)
5Ø2Ø  FOR T (2)= . 1 TO 36Ø . 1 STEP 15:  GOSUB PROG 1 :
      PRINT USING " # # # # " ; T(2)
5Ø3Ø  T (7) = T (3) + 18Ø - T (2) + T (6):  W (Ø) = W (3) - W (2)
5Ø4Ø  GOSUB PROG 3: PRINT USING "# # # # # # " ; T (7);
      W (Ø) ; T (9) ; W ;(4) ; -W (2) ; T (6) + T (4) - T (2) :
      NEXT T (2)
```

T (2)	T (7)	W (Ø)	T (9)	W (4)	W (4) *	T (4) *
Ø	4Ø5	-53	45	-35	-35	45
15	383	-52	3Ø	-35	-35	3Ø

* Computed by inversion formulas.

(see the Print statement in step 5040).

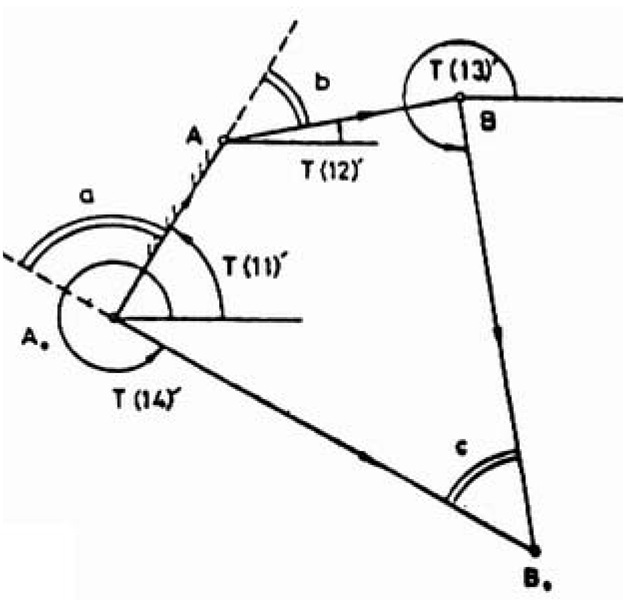
The two sets of results are seen to be identical. Figure 7 shows the variation of the crank angle in the FISC with the crank angle in the slider-crank mechanism.

It is important to observe that further analyses can now be carried out by the use of kinematic coefficients for different input speeds and also for input from different members.

KINEMATIC ANALYSIS OF A FOUR-BAR LINKAGE

The four-bar linkage is analyzed below for position, velocity, and acceleration to illustrate the use and flexibility of the method of inversion.

Figure 9: The four barlinkage with AoA fixed.



Example 2: Given for the four-bar linkage shown Figure 8 that AoBo=5, AoA=2, AB= 6, BoB=4 cm, and a uniform tw crank speed of 9 rad/s at AoA, it is desired to find T(13) (Figure 9) and its derivatives if AoA is fixed (Figure 8) such that the inclination of AoA is 60°.

Solution: The three relative angles selected for comparison are shown in Figures 8 and 9 by double arcs. Equating each relative angle in Figure 8 with the corresponding angle in Figure 9,

T(11) - T(12) = T(14)' - 180 - - T(11) [3]
T(12)- T(13) = T(11)' - [T(12)' [4]
T(11) - T(14) = T(14)' - 180 - [T(13'-180] [5]

Note in Eqns 3 to 5 that T(11), T(12), T(13), T(14) and their derivatives are either given, or can be determined by calling Prog 2 of AL-YASEER. Since T(11)' is likewise given,

T(14)' = T(11) - T(12) + T(11)' + 180^0 [6]

and, taking derivatives [8]

W(7)'= -W(5) [7]
A(7)' = -A(5) [8]

Proceeding similarly for Eqns 4 and 5

T(12)' = T(11)' + T(13) - T(12) [9]
W(5)'= W(6) - W(5) [10]
A(5)' = A(6) - A(5) [11]
T(13)' = K(14)' + T(14) - T(11) [12]
W(6)' = W(7)' + W(7) [13]
A(6)' = A(7)' + A(7) [14]

For convenience in programming let,

B(1) = T(11) = 165B(5) = W(5) = 9   B(8) = A(5)

Table 2: Analysis of a Four-Bar Linkage by Inversion.

```
6Ø1Ø  FOR QQ = . 1 TO 36Ø . 1 STEP 2Ø : T(12) = QQ : L (11) = 5 : T (11) = 165:  L (12) = 2 : W (5) = 9 : L (13) = 6 : L (14) = 4 :
      GOSUB PROG 2
6Ø2Ø  B (1) = 165:   B (2) = T (12) : B (3) = T (13) :   B (4) = T (14) : B (5) = W (5) : B (6) = W (6) : B (7) = W (7) : B (8) = A (5) : B (9) =
      A (6) : B (1Ø) = A (7)
6Ø3Ø  B (14) = B (1) - B (2) + 6Ø + 18Ø :   B (17) = - B (5) : B (2Ø) = - B (8) : B (12) = 6Ø + B (3) - B (2) :   B (15) = B (6) - B (5) :   B
      (18) = B (9) - B (8)
6Ø4Ø  B (13) = B (14) + B (4) - 165 :   B (16) = B (17) + B (7) :   B (19) = B (2Ø) + B (1Ø) : PRINT USING "# # # # # # " ; QQ ; B (13) ;
      B (16) ; B (19) ;
61ØØ  L (11) = 2 :  T(11) = 6Ø : L(12) = 6:  L (13) = 4 : L (14) = 5 :  T (12) = B (12) : W (5) = B (15) : A (5) = B (18) : GOSUB PROG 2
611Ø  PRINT USING "# # # # # " ; T (13) ; W (6) ; A (6) : NEXT QQ : END
```

T (12)	T (13) '	W (6) '	A (6) '	T (13) '	W (6) '	A (6) '
Ø	281	-11	13 Ø	-79	-11	13Ø
2Ø	261	-7	18	261	-7	18

B(2) = T(12) B(6) = W(6) B(9) = A(6)
B(3) = T(13) B(7) = W(7) B(10) = A(7)
B(4) = T(14)
B(11) = T(11)' = 60B(15) = W(5)' B(18) = A(5)'
B(12) = T(12)' B(16) = W(6)' B(19) = A(6)'
B(13) = T(13)' B(17) = W(7)' B(20) = A(7)'
B(14) = T(14)'

Casting the relationship 6-14 in the form of a program (Table 2), we compute B(11) to B(20) (steps 6030-6040), after first calling Prog 2 of AL-YASEER to determine B(3) to B(10) (steps 6010-6020).

If, for the mechanism of Figure 9, the data for T(13)', W(5)' and A(5)' is supplied, ascomputed by Eqns 9-11 above, and Prog 2 is called (step 6100), one would expect to obtain the same results as those computed for B(11) to B(10). This, indeed is the case, as may be verified from columns 5-7 of Table 2.

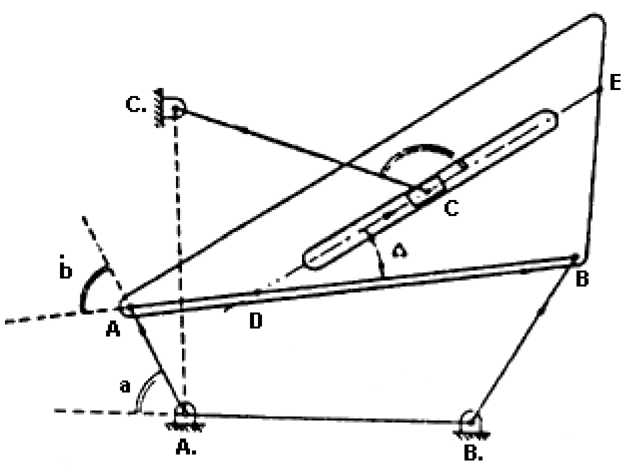


Figure 10: Example of a linkage with movable slider axis.

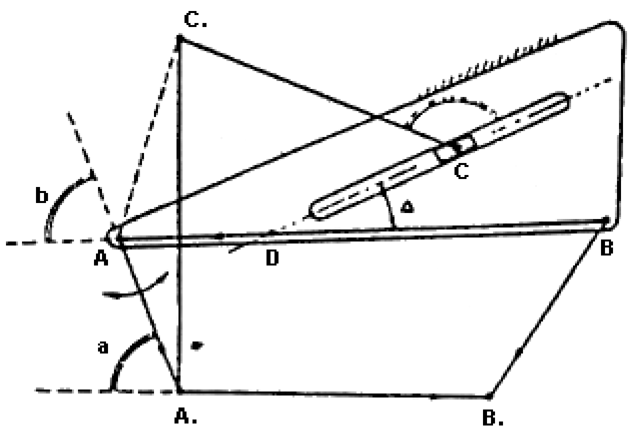


Figure 11: The mechanism of Figure 10 with coupler AB fixed.

APPLICATIONS TO COMPLEX MECHANISMS

Consider Figure 10 as an illustration of a complex mechanism, where the geometry as well as the motion of the crank AoA may be known, and the motion of the output link CoC desired. Calling Prog 2 will determine the kinematics of AoABBo, including the motion of the slider axis DE, which makes a constant angle δ with respect to AB. In an effort to analyze this mechanism, we may try an inversion, and fix the coupler AB (Figure 11).

When in Figure 11 the crank AoA is given a known motion, Prog 2 will yield the motion of the four-bar AAoBoB as well as that of point Co. Thus a virtual link can be computed between A and Co. Then a regular slider-crank ACoC results, which may be studied with the aid of Prog 1. Observation of the equality of the relative displacements in the two figures, followed by the use of kinematic coefficients (15) yields the required results.

Consider next a mechanism with two sliders; a modi-

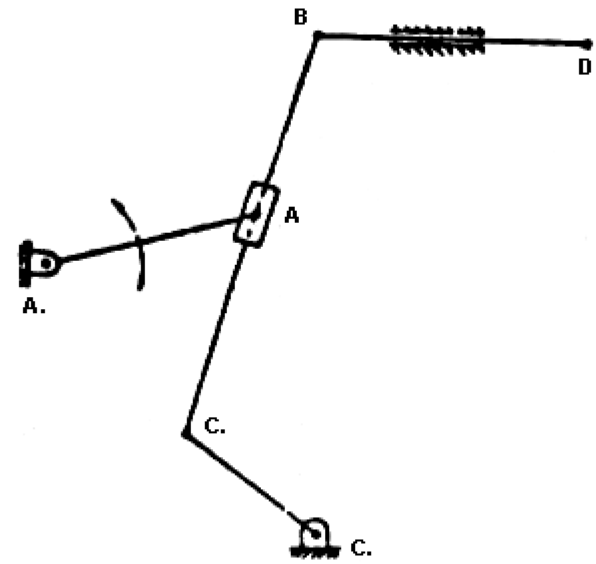


Figure 12: A modified shaper mechanism of high complexity.

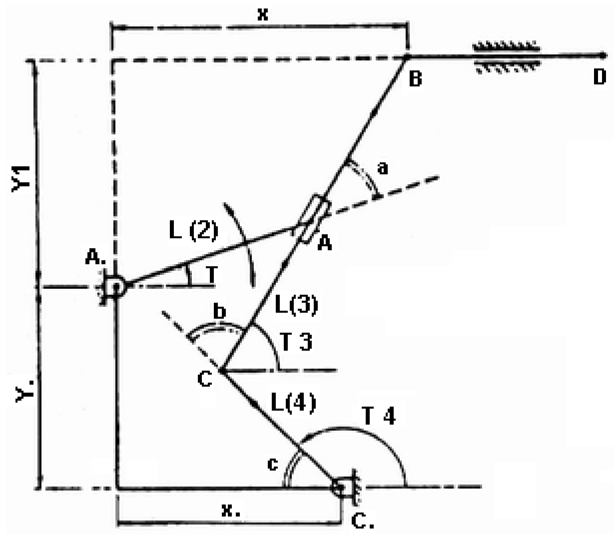


Figure 14: The modified shaper mechanism.

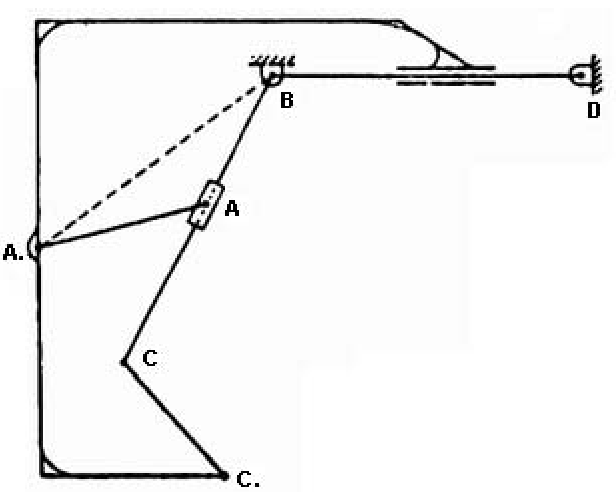


Figure 13: The mechanism of Figure 12 with one slider fixed.

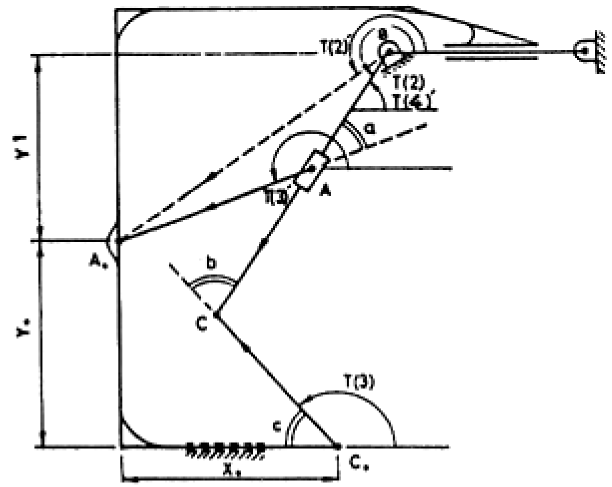


Figure 15: The modified shaper mechanism after inversion.

fied shaper mechanism of high complexity (Figure 12). One way to approach this mechanism would be to fix the horizontal slider, as shown in Figure 13. Take notice that CoAo in Figure 13 translates. Point Co behaves like a slider with a horizontal cylinder axis. Thus calling Prog 1 for BCCo wil yield the motion of Ao as well. An imaginary link BAo can be computed, which becomes the crank of a second slider-crank BAoA, where BC assumes the role of a movable-axis guide for the slider A.

Example 3: Crank AoA of the modified shaper mechanism shown in Figure 14 rotates cw at a uniform speed of 1.3 rad/s. Given that Yo = 1, Y1= 5, Xo = 4, L(2)=2, L(3)=16, and L(4)=3, it is required to find a) X, T3 T4 as functions of the crank angle T, and b) the velocity and acceleration of slider BA as well as the angular velocity and acceleration of member BC.

Solution: Figure 15 is marked in accordance with the above discussion on the same mechanism. Identifying the relative angles in Figures 14 and 15 with double arcs, and equating the corresponding relative angles,

180 - T4 = 180 - T(3) [a]

T4 - T3 = T(3) - [T(2) - 180] [b]

T3 - T = T(2) - T(3)' [c]

Since values for T(2), T(3) and T(3)' are known (Figure 15),

T4 = T(3)

T3 = T4 + T(2) - T(3) - 180
T = T3 + T(3)' - T(2)

Upon rearranging,
T4 = T(3) [15]
T3 = T(2) - 180 [16]
T = T(3)' - 180 [17]
Differentiation of Eqns 15-17 yields,
W4 = W(3) [18]
A4 = A(3) [19]
W3 = W(2) [20]
A3 = A(2) [21]
W = W(3)’ [22]
A = A(3)' [23]

Procedure: Call Prog 1 for BCCo for 190 < T(2) < 350° as an initial trial, with a step size 10^0. This is to spot the operational region of the mechanism. Then refine your limits. In the present case these limits are found to be 266 < T3 < 286^0 (Figure 14).

Next define the virtual link [7] BAo
X(10) = X(2) - X0
Y(10) = Y(2) + Y0
and call Prog 5 of AL-YASEER. Then call Prog 1 for the slider-crank with variable slider axis, the BAoA, noting that L(1) here is zero. Print the results for displacements.

Take notice of the fact that T(4)' will have two values for every value of T(2) (Figure 15). These will be 180° apart, and will depend on the orientation of AoA. Table 3 lists the program and a sample of the results. Steps 3010 (KK=180) and 3080 help locate the two positions. This explains why values of the dummy variable QQ are listed in duplicate. Figure 16 displays the variation of displacements with crank angle.

Table 3: Displacement Analysis of Shaper Mechanism of High Complexity.

```
3Ø1Ø  YØ = 1Ø : Y1 = 5 : XØ = 4 : L2 = 2 : L3 =16 : L4 = 3 : KK = 18Ø
3Ø2Ø  FOR QQ = 260 TO 286 STEP 2 : T(2) = QQ : L (1) = Y1 + YØ : T (4) = . 1:
      L (2) = L3 : L(3) = L4 : GOSUB PROG 1 : CLS
3Ø3Ø  TQ = T (2) : PRINT USING “# # # # # # . # ” ; QQ ; T (3) ; T (2) - 18Ø
3Ø4Ø  X (1Ø) = X (2) - XØ :  Y (1Ø) = Y (2) + YØ : GOSUB   PROG 5
3Ø5Ø  L (2) = L : T (2) = T :  L (1) = Ø : T (4) = T Q + KK :  L (3) = L (2) : GOSUB PROG
      1 : CLS : PRINT USING “# # # # # # . # ” ; T (3) - 18Ø ;   -X (1Ø ) : NEXT   QQ :
3Ø8Ø  1F KK = 18Ø  THEN KK = Ø  : GOTO  3Ø2Ø
3Ø9Ø  END
```

QQ	T4	T3	T	X .
266	198.7	86.Ø	12.1	2.3
268	199.4	88.Ø	37.1	1.7

Table 4: Kinematics of the Shaper Mechanism.

```
3Ø1Ø  YØ = 1Ø :   Y1 = 5 :   XØ = 4 :   L2 = 2 :    L3 =16 :   L4 = 3 :   K1 = 18Ø
3Ø2Ø  FOR QQ = 266 TO 286 STEP 2 :   T(2) = QQ :   L (1) = Y1 + YØ :    T (4) = . 1:
       L (2) = L3 :   L(3) = L4 :   W (2) = 1:   D (1) = Ø : E (1) = Ø : GOSUB PROG 1 : CLS TQ = T (2)
3Ø3Ø  TR = W (2) : PRINT USING “# # # # # # . # ” ; QQ ; T (3) ; T (2) - 18Ø
3Ø35  V (18) = Ø :   A (5Ø) = Ø
3Ø4Ø  X (1Ø) = X (2) - XØ :  Y (1Ø) = Y (2) + YØ :  V (17) = V (3) :  A (49) = A (35) : GOSUB PROG 5
3Ø5Ø  L (2) = L : D (Ø) = D (6) : E (Ø) = E (6) :  T (2) = T :   W (2) = D (5) :   A (2) =
       E (5) : L (1) = Ø : T (4) = TQ + K1 :  D (1) = TR :   E (1) = Ø : Z (5) = 5 :
       L (3) = L2 : GOSUB PROG 1:   CLS :   D ( Ø) = Ø :    E (Ø) = Ø :   A (2) = Ø
3Ø7Ø  X (2) = -X (1Ø) :  V (3) = -V (17) :   A (35) = - A (49) :   IF V (3) = Ø THEN V (3) = . ØØØ1
3Ø8Ø  H= W (3) / TR : K= A (3) / TR ^2 :  HH = W (3) / V (3):  KK = ( A (3) - HH*A (35)) /V (3) ^2 : D
      (1) = -1.3 / H :   E (1) = -K*TR^2/H
3Ø9Ø  V(3) = -1.3/HH:  A (35) = -KK*V (3)^2/HH
31ØØ  PRINT USING ″ # # # # # #. # # ″ ; T (3) - 18Ø; D (1); E (1); V (3); A (35) :  NEXT QQ
311Ø  IF K1 = 18Ø THEN  K1 = Ø :  GOTO   3Ø2Ø
312Ø  END
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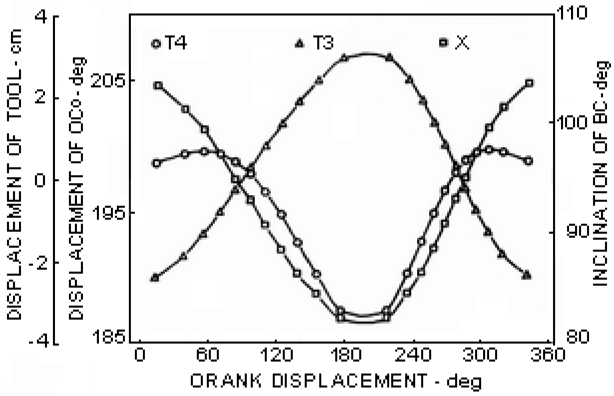

Figure 16: Variation of displacement with crank angle for the modified shaper mechanim of Example 3.

The x-coordinate of point B in Figure 15 is now-X(10) due to the change in the direction of the virtual link (step 3050). It follows hence that

VB = -V(3)

AB = -A(35)

where VB and AB denote the velocity and acceleration of B, respectively.

The kinematics part of the computation follows the usual routine, and is listed in Table 4, which is a listing of the complete program. Step 3020 resets values for the

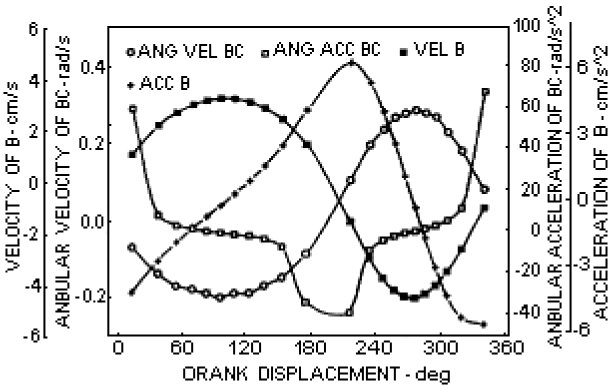

Figure 17: Kinematics of the modified shaper mechanism of high complexity.

first slidercrank where (Figure 5) is fixed, the crank length constant and the input speed is uniform. Step 3030 saves the value of W(2). Virtual link kinematics are introduced in steps 3035-3050. D(1) and E(1) for the variable T(4) of the second slider-crank are specified in step 3050.

Step 3070 identifies initial results, with BC as input. Kinematic coefficients are calculated in step 3080. Desired velocities with AoA as the crank are computed in

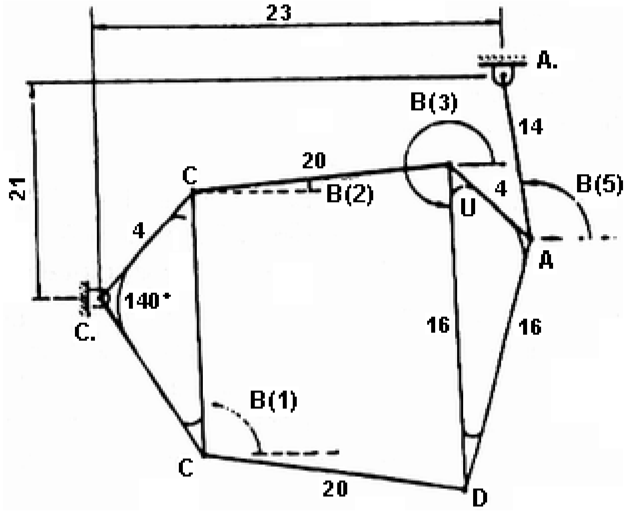

Figure 18: The CM - 3 mechanism of Example 4.

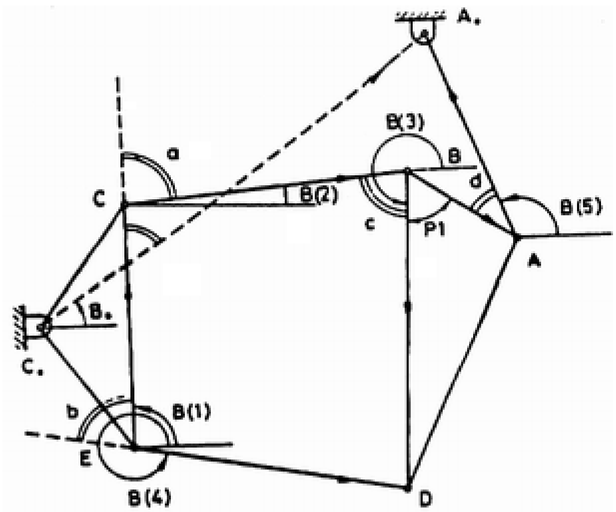

Figure 19: Relative angles in the CM - 3.

step 3090. The second half of the cycle is initiated by step 3110, as before.

The results are summarized in Figure 17.

Example 4: The CM-3 linkage shown in Figure 18 is actuated by member CoCE. Conduct a position analysis, and determine the variation of B(2), B(3), and B(5) with B(1).

Procedure: Imagine that AoCo is not the frame Figure 18. Instead, if member CoCE is fixed, and the mechanism is driven from BC, the four-bar ECBD that results is analyzable by Prog 2. Using now the virtual link CA, the motion of the remaining four-bar CAAoCo may be computed.

The relative angles in the desired (unknown) configuration are shown in Figure 19.

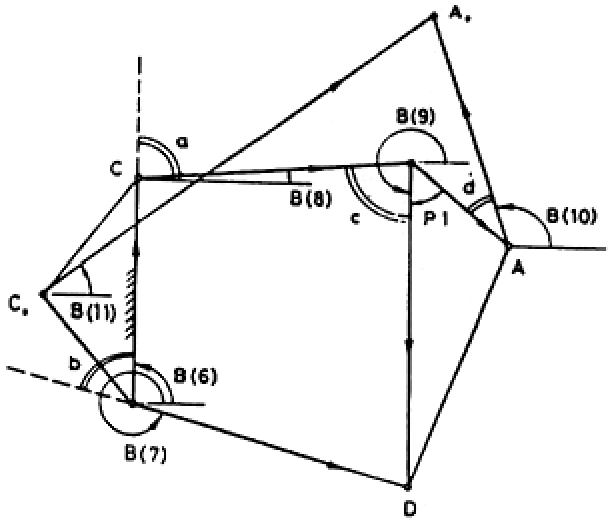

Figure 20: Relative angles in the inverted CM - 3.

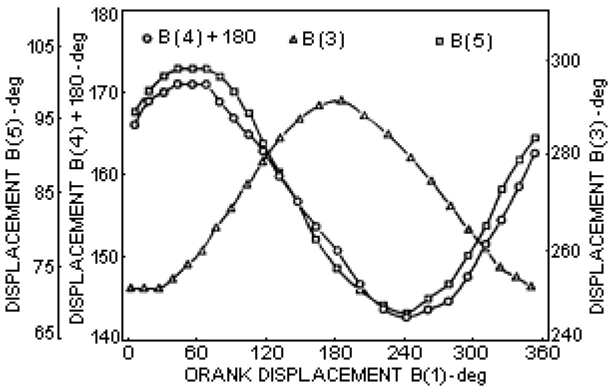

Figure 21: Displacement of the CM- 3 mechanism.

In the inverted configuration the same relative angles become as shown in Figure 20. Equating the two sets of relative angles, and noting that b(6) to B(11) as well as $\Phi 1$=P1 and B(0) = Bo are known,

$B(1) - B(2) = B(6) - B(8)$ [24]

$B(3) - B(2) - 180 = B(9) - B(8) - 180$ [25]

$B(3) + \Phi 1 + 180 - B(5) = B(9) + \Phi 1 + 180 - B(10)$ [26]

$B(4) - 180 - B(1) = B(7) - 180 - B(6)$ [27]

$B(1) - B(0) = B(6) - B(11)$ [28]

Thus

$B(1) = B(6) - B(11) + B(0)$ [29]

$B(2) = B(1) + B(8) - B(6)$ [30]

$B(3) = B(9) - B(8) + B(2)$ [31]

$B(4) = B(7) - B(6) + B(1)$ [32]

$B(5) = B(3) + B(10) - B(9)$ [33]

Utilizing the data supplied with Figure 18 on Figure 20, and letting $B(6)=90.1^0$, the input information for the first four bar is provided (steps 20- 30 in Table 5). The virtual link data as well as the information for the second four-bar are provided in steps 40-60. It is also noted that B(0)= ATN (21/23) from Figure 18.

Figure 21 displays the output. Note how short and straight forward the program of Table 5. A comparable program that uses the loop closure approach would be considerably longer in length, and in solution time.

Example 5: It is desired to make a kinematic analysis of the problem of Example 4 (Figure 19) when member BC of Figure 20 has a uniform cw speed of 1.0 rad/s.

Solution: Referring to Eqns 29-33 of Example 4, and letting

$B(12) = dB(1)/dt$	$B(13) = d^2 B(1)/dt^2$
$B(14) = dB(2)/dt$	$B(15) = d^2 B(2)/dt^2$
$B(16) = dB(3)/dt$	$B(17) = d^2 B(3)/dt^2$
$B(18) = dB(4)/dt$	$B(19) = d^2 B(4)/dt^2$
$B(20) = dB(8)/dt$	$B(21) = d^2 B(8)/dt^2$
$B(22) = dB(9)/dt$	$B(23) = d^2 B(9)/dt^2$
$B(24) = dB(10)/dt$	$B(25) = d^2 B(10)/dt^2$
$B(26) = dB(7)/dt$	$B(27) = d^2 B(7)/dt^2$
$B(28) = dB(5)/dt$	$B(29) = d^2 B(5)/dt^2$
$B(30) = dB(0)/dt$	$B(31) = d^2 B(0)/dt^2$
$B(32) = dB(11)/dt$	$B(33) = d^2 B(11)/dt^2$

we obtain, after observing that $dB(6)/dt = d^2hB(6)/dt^2 = 0$,

$B(12) = -B(32) + B(30)$ [34]

$B(13) = -B(33) + B(31)$ [35]

$B(14) = B(12) + B(20)$ [36]

$B(15) = B(13) + B(21)$ [37]

$B(16) = B(22) - B(20) + B(14)$ [38]

$B(17) = B(23) - B(21) + B(15)$ [39]

$B(18) = B(26) + B(12)$ [40]

$B(19) = B(27) + B(13)$ [41]

$B(28) = B(16) + B(24) — B(22)$ [42]

$B(29) = B(17) + B(25) - B(23)$ [43]

Figure 22: Kinematics of CM - 3.

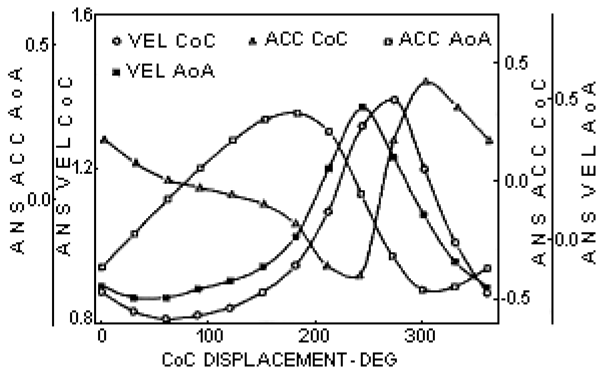

Table 5: Inversion Analysis of CM - 3 (Displacements).

```
2Ø  FOR QQ = 36Ø .1 TO Ø.1 STEP -15: L(1Ø) = Ø : T(11) = 9Ø . 1 : L (11) = SQR (32 -
    32*COS14Ø): T (12) = QQ : L (12) = 2Ø : L (13) = 16 : L (14) = 2Ø
3Ø  R (11) = 4: H (11) = ACS (16 / 16 / 8) : GOSUB PROG 2
4Ø  B (6) = 9Ø.1 : B (8) = T (12) : B (9) = T (13) : B (7) = T (14)
5Ø  X (1Ø) = X (4) : Y (1Ø) = Y (4) : GOSUB PROG 5 : L (12) = L : T (12) = T
6Ø  L (1Ø) = 1 : L (11) = 4 : T (11) = 7Ø : L (13) = 14 : L (14) = SQR (21^2 + 23^2)
7Ø  GOSUB PROG 2 : B (1Ø) = T (13) : B (11) = T (14) : B (Ø) = ATN (21 / 23) : B (1) = B (6)
    - B (11) + B (Ø) : B (2) = B (1) + B (8) - B (6)
8Ø  B (3) = B (9) - B (8) + B (2) : B (4) = B (7) - B (6) + B (1) : B (5) = B (3) + B (1Ø) - B (9) :
PRINT USING " # # # # # # " ; QQ ; B (1) ; B (4) + 18Ø ; B (3) ; B (5)
9Ø  NEXT QQ : END
```

Table 6: Inversion Analysis of CM - 3 (Displacements).

```
2Ø  FOR QQ = 36Ø .1 TO Ø.1 STEP -15: L(1Ø) = Ø : T(11) - 9Ø . 1 : L (11) = SQR (32 -
    32*COS14Ø): T (12) = QQ : W (5) = - 1 : L (12) = 2Ø : L (13) = 16 : L (14) = 2Ø R (11) = 4 :
    H (11) = ACS (16 / 16 / 8) : GOSUB PROG 2
25  B (6) = 9Ø.1 : B (8) = T (12) : B (9) = T (13) : B (7) = T (14) : B (2Ø) = W (5) : B (21) = A (5) : B
    (22) = W (6) : B (23) = A (6) : B (26) = W (7) : B (27) = A (7) 3Ø X (1Ø) = X (4) : Y (1Ø) = Y (4) :
    V (17) = V (7) : V (18) = V (8) A (49) = A (39) : A (5Ø) = A (4Ø) : GOSUB PROG 5
4Ø  L (12) = L : D (2) = D (6) : E (2) = E (6) : T (12) = T : W (5) D(5) : A (5) - E (5) : L (1Ø) = 1 :
    L (11) = 4 : T (11) = 7Ø : L (13) = 14 : L (14) = SQR (21^2 + 23 ^ 2) : GOSUB PROG 2 : D (2)
    = Ø : E (2) = Ø : A (5) = Ø
6Ø  B (1Ø) = T (13) : B (24) = W (6) : B (25) = A (6) : B (11) = T (14) : B (32) = W (7) : B (33) = A (7) :
    B (Ø) = ATN (21 / 23) : B (1) = B (6) - B (11) + B (Ø)
7Ø  B (2) = B (1) + B (8) - B (6) : B (3) = B (9) - B (8) + B (2) : B (4) = B (7) - B (6) + B (1) : B (5) = B (3) +
    B (1Ø) - B (9)
8Ø  B (12) = - - B (32) + B (3Ø) : B (13) = - B (33) + B (31) : B (14) = B (12) + B (2Ø) : B (15) - B (13)
    + B (21) : B (16) = B (22) - B (2Ø) + B (14)
9Ø  B (17) = B (23) - B (21) + B (15) : B (18) = B (26) + B (12) : B (19) = B (27) + B (13) : B (28) = B
    (16) + B (24) - B (22) : B (29) = B (17) + B (25) - B (23)
1ØØ PRINT USING " # # # # # . # # " ; QQ ; B (1) - 2Ø ; B (5) ; B (12) ; B (13) ; B (28) ; B (29) :
    NEXT QQ : END
```

Incorporation of Eqns 34 - 43 into the program of Table 5 results in the program shown in Table 6. The corresponding results are summarized in Figure 22.

Example 6: It is desirable to determine the angular velocity and acceleration of member AoA of the mechanism of Example 5 (Figure 19) when the driver link is CCoE, and it has a uniform clockwise velocity of 10 rad/s,

Solution: Recalling from the method of kinematic coefficients that

$w_{AoA} = H * w_{CoC}$ [44]

$\alpha_{AoA} = Kw_{CoC}\text{^}2 + H * \alpha_{CoC}$ [45]

for the known values of these at each step (computed in Example 5) the instantaneous value of H and K may be found:

$H = w_{AoA} / w_{CoC}$ [46]

$K = [\alpha_{AoA} — H * \alpha_{CoC}] / w_{CoC}\text{^}2$ [47]

Then to compute any new value of w_{CoC} and α_{CoC} at the same position, Eqns 44 and 45 may be utilized. In this particular

case $w_{AoA} = -10*H$ [48]

$\alpha_{AoA} = 100*K$ [49]

Eqns 46-49, when cast in the symbolism of the current program (Table 7), become

H= B(28) / B(12)

K= [B(29) - H*B(13)] / B(12)^2

b(28) = -10*H

B(29) = 100*K

Table 7: Kinematics of CM- 3 in Example 6.

```
2Ø   FOR  QQ = 36Ø .1 TO  Ø.1  STEP   -15:  L(1Ø) = Ø :    T(11) = 9Ø . 1 :    L (11) =
     SQR  (32 - 32* COS14Ø):  T (12) = QQ :   W (5) = - 1 :    L (12) = 2Ø :    L (13) = 16 :
     L (14)  =  2Ø :    R (11) = 4 :    H (11) = ACS (16 / 16 / 8) :    GOSUB PROG 2
25   B (6) = 9Ø.1 :    B (8) = T (12) :    B (9) = T (13) :    B (7) = T (14) :  B (2Ø) = W (5) :
     B (21) = A (5) :  B (22) = W (6) :   B (23) = A (6) :   B (26) = W (7)  : B (27) = A (7)
3Ø   X  (1Ø) =  X (4) :  Y (1Ø) = Y (4) :   V (17) =  V (7)  :  V (18) =  V (8)  A (49)  =   A (39)
     :  A (5Ø)  =  A (4Ø)  :   GOSUB PROG 5
4Ø   L (12) =  L :   D (2) = D (6) :   E (2) =  E (6) :   T (12) = T  :  W (5)   D(5)  :  A (5) =
     E (5) :  L (1Ø) = 1 :    L (11) =  4  :   T (11) = 7Ø :   L (13) = 14 :  L (14) = SQR (21 2
     +  23 ^ 2) : GOSUB PROG 2 :   D (2) = Ø :    E (2) = Ø  :   A (5) = Ø
6Ø    B (1Ø) = T (13) :   B (24) = W (6) :   B (25) = A (6) :   B (11) = T (14) :   B (32) = W (7)
      :  B (33) = A (7) :    B (Ø) =  ATN (21 / 23) :  B (1) = B (6) - B (11) + B (Ø)
7Ø    B (2) =  B (1) + B (8) - B (6) :  B (3) = B (9) - B (8) + B (2) :  B (4) = B (7) - B (6) +
      B (1) :   B (5) = B (3) + B (1Ø)  -  B (9)
8Ø    B (12) =  - B (32) + B (3Ø)  :  B (13) = - B (33) +  B (31)  :  B (14) = B (12) +  B (2Ø) :
      B (15) - B (13) + B (21) : B (16) = B (22) -  B (2Ø)  +  B (14)
9Ø    B (17) = B (23) - B (21)  +  B (15)  :  B (18) = B (26)  + B (12) :  B (19) = B (27)  +
      B (13) :  B (28) = B (16) +  B (24) - B (22) :  B (29) = B (17) + B (25)  -  B (23)
1ØØ  H = B (28) / B (12) :    K = (B (29) - H * B (13)) / B (12) ^2 :    B (28) = -1 Ø * H:
      B (29) = K * 1ØØ
11Ø PRINT USING  ″ # # # # # . # # ″ ; QQ ; B (1) -  2Ø ; B (5) + 18Ø ; B (28) ;  B (29) :
      NEXT  QQ :  END
```

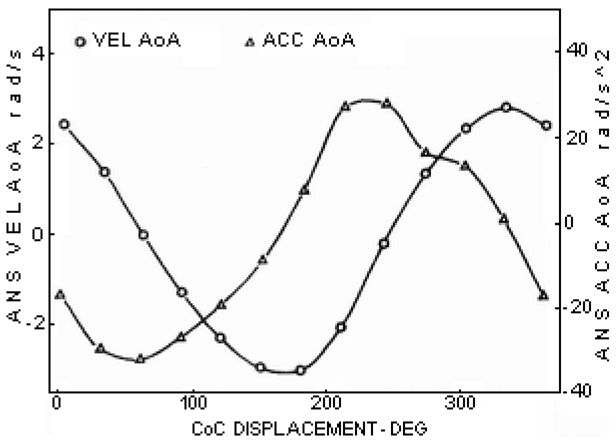

Figure 23: Motion of the output link in CM - 3 of Example 6.

Figure 23 persents the variation of B(28) and B(29) with crank angle. Suffice it to state that the curves b(28) and B(29) display the results of exact calculations; no approximations have been introduced.

DISCUSSION

It may be stated in general that, despite all its power and convenience, the method of kinematic coefficients is not always adequate to handle the analysis of mechanisms of high complexity. The method of inversion of the present article ushers in possibilities of far-reaching consequences in this respect.

It may be that a certain selected mechanism is not analyzable by a direct application of AL-YASEER. Whenever such a mechanism is confronted, it would be proper strategy to try to change the driving member, and thus analyze the mechanism. In many cases this leads to the transformation of the mechanism such that it becomes readily solvable by AL-YASEER. The resulting output then can be converted into the output of the original mechanism by the aid of the method of kinematic coefficients.

Should the case arise where the above strategy fails, that is, the mechanism is not cast into a form solvable by AL-YASEER even after changes in the drive member, then it is time to try an inversion. Inversion implies the transformation of the mechanism such that the frame member is replaced. When this is done, and a member adjoining the new fixed member is made the driver, it is generally found that the resulting mechanism can be readily analyzed. The output from this analysis can be translated into the output of the original mechanism by setting up simple correspondence rules.

Once the output for the original mechanisms found by the method of inversion, the method of kinetic coefficients can be made use of to obtain results at different input kinematics, and also for input from a different member. The effort involved and the resulting solution time may be favorable when compared with solution by the loop-closure technique.

ACKNOWLEDGEMENT

This work is partially based on research related to KAU Project No 407-090, Jeddah.

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