A mathematical approach to mitral regurgitation

Mitral regurjitasyona matematiksel bir yaklaşım

Left ventricular end-diastolic pressure, left atrial pressure (LAP) and pulmonary capillary wedge pressure has been the subject of many investigations. The reason is that these pressures provide a good guide to assess the heart failure. Echocardiography is the method of choice in many instances. Evaluation of left ventricular pressure (LVP) may help in echocardiographic analysis of left ventricular filling pressures.

The mitral regurgitation (MR) can be detected by echocardiography with nearly 100% sensitivity (1). The MR is a good guide to assess left ventricular pressure. The Doppler recording of MR (MR jet) depends on the pressure gradient between left ventricle and left atrium, because it is this gradient that pushes the blood from left ventricle to left atrium. The relation has been described as (2):

$\Delta P = 1/2\delta(V_2^2 - V_1^2) + \delta(dV/dt)*ds + R(\mu)$ (Eq. 1)

where ΔP is the pressure difference across the stenosis, V1 and V2 are the velocities proximal and distal to the stenosis, respectively, δ is the mass density of the blood, R is viscous resistance and μ is viscosity. The last two terms are negligible; therefore, the Eq-1 converts to:

$\Delta P = 4\ (V_2^2 - V_1^2)$ (Eq. 2)

In cases of very low V1, the equation may finally be converted to:

$\Delta P = 4\ V_2^2$ (Eq. 3)

The MR jet is in fact a graphic of blood velocity versus time. Using the relation between ΔP and blood velocity, we can rewrite the Eq. 3 as:

$\Delta P/4 = V^2$ (Eq. 4)

We know that:

$\Delta P = LVP - LAP$ (Eq. 5)

where LVP is the left ventricular pressure and LAP is the left atrial pressure. Then,

$LVP = \Delta P + LAP$ (Eq. 6)

During ventricular systole, change in LVP is much more prominent than change in LAP, with the exceptions of presence of V-wave. With this assumption, it can be concluded that the MR jet is also a reflection of LVP.

Although the entire time course of LVP cannot be converted to certain equations because of the variability in aortic properties, some equations have been proposed for isovolumic contraction and isovolumic relaxation phases, which are the periods that the aortic valve is not open, and therefore, aortic pressure is not equal to the left ventricular pressure. The mostly accepted example of such an equation is Weiss' formula which was suggested for defining left ventricular pressure during isovolumic relaxation phase (3):

$P = P_0 \times e^{-t/\tau}$ (Eq. 7)

where the P is the instant pressure during isovolumic relaxation phase, P_0 is the left ventricular pressure at any time before the time of left ventricular pressure of P, e is the base of natural logarithm, t is the time interval between the pressure of P_0 and P, τ is the isovolumic relaxation time constant.

Actually, P is equal to ΔP + LAP and P_0 is equal to ΔP_0 + LAP. So, we can convert the equation (Eq. 7) to another equation:

$(\Delta P + LAP) = (\Delta P_0 + LAP) \times e^{-t/\tau}$ (Eq. 8)

We can give some absolute numbers for ΔP and ΔP_0, such that $\Delta P = 0$ mmHg and $\Delta P_0 = 64$ mmHg or $\Delta P_0 = 36$ mmHg (Fig. 1). So we can rewrite the equations as:

$(0 + LAP) = (64 + LAP) \times e^{-t1/\tau}$ (Eq. 9)

$(0 + LAP) = (36 + LAP) \times e^{-t2/\tau}$ (Eq. 10)

Natural logarithm of each side of each equation gives two other equations:

$\ln(LAP) = \ln(64 + LAP) - t1/\tau$ (Eq. 11)

$\ln(LAP) = \ln(36 + LAP) - t2/\tau$ (Eq. 12)

We can convert these equations into two other equations:

$t1/\tau = \ln(64 + LAP) - \ln(LAP)$ (Eq. 13)

$t2/\tau = \ln(36 + LAP) - \ln(LAP)$ (Eq.14)

If we divide right side of the Eq. 13 with the right side of the Eq. 14 and left side with left side, we obtain another equation:

$t1/t2 = [\ln(64 + LAP) - \ln(LAP)] / [\ln(36 + LAP) - \ln(LAP)]$ (Eq. 15)

It is seen that t1/t2 is a function of LAP alone. This hypothesis was tested by a clinical study and it was confirmed that this principle is valid (4).

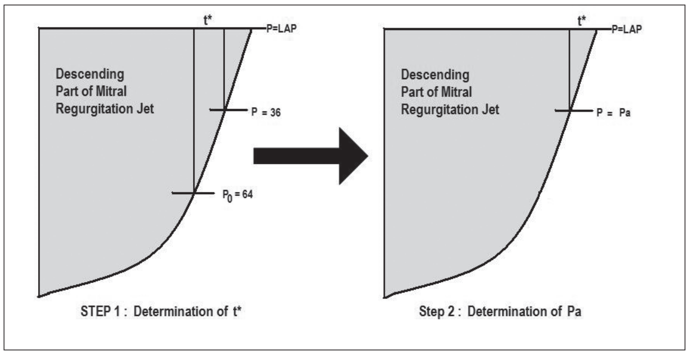

Figure 1. The illustration of determining t* and Pa

There is another support for the validity of the Eq. 8. It was proposed by Chen et al. (5) that τ can be measured from the MR jet. Their method was based on the measurement of the time duration from 3 m/s velocity to 1 m/s velocity of MR jet on the descending part. So they used the ΔP as 36 mmHg and 4 mmHg. Then:

$(LAP + 4) = (LAP + 36) \times e^{-t/\tau}$ (Eq. 16)

According to their proposal, the t should be equal to τ, then:

$(LAP + 4) = (LAP + 36) \times e^{-\tau/\tau}$ (Eq. 17)

$(LAP + 4) = (LAP + 36) \times e^{-1}$ (Eq. 18)

$(LAP + 4) = (LAP + 36) \times 0{,}37$ (Eq. 19)

According to this equation, LAP should be 14.8 mmHg to validate this method. However, they have reported that they had assumed LAP as 10 mmHg. On the other hand, they also confessed that this assumption has led to underestimation of the τ. Although they have not mentioned, the underestimation of τ may be explained by this assumption. Never the less, this assumption may take us to another position. We can easily measure the time interval from 3 m/s to 1 m/s on the MR jet (Fig. 1, step 1). Let's define this interval as t*. Afterwards, let's find the point on the descending part of the jet, which is t* away from the end of the jet. Let's define this point as Pa (Fig. 1, step 2). Then:

$(0 + LAP) = (Pa + LAP) \times e^{-t^*/\tau}$ (Eq. 20)

$(4 + LAP) = (36 + LAP) \times e^{-t^*/\tau}$ (Eq. 21)

If we divide one side of the Eq. 22 with same side of the Eq. 23, then:

$LAP/(4 + LAP) = (Pa + LAP) / (36 + LAP)$ (Eq. 22)

Then,

$LAP = (4 \times Pa) / (32 - Pa)$ (Eq. 23)

It is seen from the equation that Pa is a function of LAP and gives information about the LAP.

We can convert Eq. 8 to another equation:

$e^{-t^*/\tau} = (36 + LAP) / (64 + LAP)$ (Eq. 24)

Natural logarithm of each side of the equation is:

$-t^*/\tau = \ln[(36 + LAP) / (64 + LAP)]$ (Eq. 25)

$t^*/\tau = \ln[(64 + LAP) / (36 + LAP)]$ (Eq. 26)

If we substitute LAP with the one in Eq. 23, then:

$t^*/\tau = \ln[(64 + ((4 \times Pa) / (32 - Pa)) / (36 + ((4 \times Pa) / (32 - Pa)))]$ (Eq. 27)

So we can also conclude that, t*/τ is a function of Pa. Therefore, if we can accurately point the Pa, we will obtain information about two important parameters of diastolic function: LAP and τ.

In fact, it is very difficult to use these equations for exactly predicting the parameters because they are very complex. However, after proving the validity of these hypotheses, the devices can automatically calculate the dependent parameters, resulting in more efficient use of Doppler echocardiography.

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References

1. Bolca O. An overview of the mitral regurgitation. Anadolu Kardiyol Derg 2004; 4: 8-9.
2. Feigenbaum H. Armstrong WF, Ryan T. editors. Feigenbaum's Echocardiography. Philadelphia. Lippincott Williams and Wilkins. 2005.
3. Weiss JL, Frederiksen JW, Weisfeldt ML. Hemodynamic determinants of the time-course of fall in canine left ventricular pressure. J Clin Invest 1976; 58: 751-60.
4. Uzun M, Erinç K, Kırılmaz A, Baysan O, Sağ C, Kılıçaslan F, et al. A Novel method to estimate pulmonary artery wedge pressure using the downslope of the Doppler mitral regurgiant velocity profile. Echocardiography 2004; 21: 673-9.
5. Chen C, Rodrigues L, Levine RA, Weyman AE, Thomas JD. Noninvasive measurement of the time constant of the left ventricular relaxation using the continuous-wave Doppler velocity profile of mitral regurgiation. Circulation 1992; 86: 272-8.

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Myxomanın arteri

The myxoma's artery

Sayın Editör,

Kardiyak miksomalar en sık rastlanan kardiyak neoplazilerdir. Tüm kardiyak neoplazilerin %30 ile 50'sini oluştururlar. Anjiyografik olarak kanıtlanmış neovaskülarizasyon %30 ile 40'ında görülmektedir. Neovaskülarizasyon sol sirkumfleks veya sağ koroner arterden eşit sıklıkla köken alır (1).

Miksomanın besleyici arterine yazarların (2-4) ve derginizin gösterdiği ilgi bizleri memnun etmektedir. Ekibimiz de miksomanın arterinin görüntülendiği bir olgu nedeniyle bu konu ile ilgilenmektedir (5) ve katkıda bulunmayı amaçlamaktadır.

Bir koroner arter ile bir kalp boşluğu arasında bulunan fistüller genelde doğuştan olup anjiyografi için refere edilen hastaların %0,17'sinde mevcuttur. Bir koroner arter ile bir kalp boşluğu arasında akkiz fistüle nadiren rastlanır. Aterosklerotik koroner arter hastalığı, mitral stenozla birlikte sol atriyal trombüs varlığı, rezeke edilmemiş miksoma varlığında fistül rapor edilmiş olup bir atriyal miksoma rezeksiyonu sonrası sol atriyum içine fistül oluşumu daha nadirdir. Bilindiği üzere, miksoma tanısı alan her hastada koroner anjiyografi rutin olarak uygulanmamaktadır. Genelde 40 yaş altı erkekler ve 45 yaş altı kadınlarda ekokardiyografi ile tanı kesinleştirilip cerrahi tedaviye yönelinir. Anjiyografi uygulanmaz. Miksomalar, ağırlıklı olarak atriyal septumdan kaynaklanır ve cerrahi tedavisinde miksomanın pedikülü ile birlikte yaklaşım 1 cm çapında atriyal septum da rezeke edilir. Septumda oluşan defekt, sonrasında onarılır. Roth ve arkadaşlarının (1) çalışması, sol atriyumdaki miksoma rezeksiyonundan yıllar sonra bile arteriyoatriyal fistül gelişebileceğini göstermiştir. Bu makale, bizim kanaatimize göre, yukarıdaki cerrahi yöntemin gözden geçirilmesi gerekliliğini ortaya koymaktadır. Miksomanın besleyici arteri anjiyografik olarak görüntülense de görüntülenmese de, cerrahi sırasında bu arter bağlansa da bağlanmasa da miksomanın sapının etrafındaki dokunun (atriyal septum, atriyal duvar) –eğer uygunsa- radyofrekans ablasyon yöntemi ile koterize edilmesi gerekliliği kanısını oluşturmaktadır. Diğer cerrahi koterler, kullanımları halinde aritmojenik odak teşkil edebileceği için, radyofrekans ablasyon uygulamasının tercih edilmesi gerektiğini düşünmekteyiz (5).

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Kaynaklar

1. Roth JE, Conner WC, Porisch ME, Shry E. Sinoatrial nodal artery to right atrium fistula after myxoma excision. Ann Thorac Surg 2006; 82: 1106-7.
2. Selçuk H, Selçuk M. T, Özbakır CI, Çaldır V, Zengin N İ, Korkmaz Ş. Anjiyografik tümör vaskülaritesine sahip dev sol atriyal miksoma. Anadolu Kardiyol Derg 2005; 2; 155-6.
3. Yıldırım T. Sol atriyal miksomada anjiyografik olarak görülebilen neovaskülarizasyon sıklığı nedir?/ Anjiyografik tümör vaskülaritesine sahip dev sol atriyal miksoma. Anadolu Kardiyol Derg 2006; 6; 338-9.
4. Yıldırım T, Selimoğlu Ö, Çevik C, Kurtoğlu N, Oğuş N T, Dindar İ. Left atrial myxoma showing angiographic neovascularization. Anadolu Kardiyol Derg 2006: 3; 299-300.
5. Kestelli M, Lafçı B, Emrecan B, Gürbüz A. Why radiofrequency ablation in the presence of nutrient artery of myxoma. Chirurgia 2007; 20: 31-2.

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